

VOLUME 80

SEPARATE No. 437

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

MAY, 1954



DISCUSSION
OF PROCEEDINGS - SEPARATES

180, 184, 186, 189

HYDRAULICS DIVISION

*Copyright 1954 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America*

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE \$0.50 PER COPY

Current discussion of papers sponsored by the Hydraulics Division is presented as follows:

<u>Number</u>		<u>Page</u>
180	A Direct Step Method for Computing Water-Surface Profiles (Published in March, 1953. Discussion closed)	
	Steinberg, I. H.	1
	Gray, C. A. M.	7
	Buehler, Bob	8
	Kansoh, Monir M.	13
	Ezra, Arthur A. (Closure)	14
184	Dam Modifications Checked by Hydraulic Models (Published in April, 1953. Discussion closed)	
	Hickox, G. H.	17
	Harrison, E. S. and Kindsvater, Carl E. (Closure) . . .	19
186	Settling Rates of Suspensions in Solids Contact Units (Published in April, 1953. Discussion closed)	
	Anderson, Norval E.	21
	McKinney, Ross E.	22
	Kalinske, A. A. (Closure)	24
189	Snow Hydrology for Multiple-Purpose Reservoirs (Published in May, 1953. Discussion closed)	
	Linsley, Ray K.	27
	Gerdel, R. W.	28
	Hafterson, Harold D.	32
	Rockwood, David M.	34
	Riesbol, Herbert S. (Closure)	37

Reprints from this publication may be made on condition that the full title of paper, name of author, page reference (or page number), and date of publication by the Society are given.

The Society is not responsible for any statement made or opinion expressed in its publications.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-Ninth Street, New York 18, N.Y.

DISCUSSION OF A DIRECT STEP METHOD FOR COMPUTING WATER-SURFACE PROFILES PROCEEDINGS—SEPARATE 180

I. H. STEINBERG.⁴—The advantages of any graphical or semigraphical

⁴Superv. Hydr. Engr., Corps of Engrs., U. S. Dept. of the Army, San Francisco, Calif.

solution over the trial-and-error methods of flow line computations lie in the frequent use of the derived curves. Where only a relatively few profiles of given discharges and conditions are required, it has been found practicable to draw the elevation-area curves and elevation-($A R^4$) curves to a convenient scale and use the trial-and-error method for tracing the water-surface profile. There have been many occasions, however, when it was necessary for the writer to compute numerous water-surface profiles over the same stretch of river. It would have been an insurmountable task were it not for the fact that it was possible to use graphical solutions. In most of the graphical methods corrections for velocity head were not considered.

The author has presented a relatively simple graphical method of computing directly the water-surface profiles in natural and artificial watercourses. Where profiles for discharges of different magnitudes are required, it would be desirable to present the pairs of curves for the discharges on separate axes for each reach. Thus, only the curves of $Z + \phi(Z)$ for station 80 + 00 and the curves of $Z + f(Z)$ for station 85 + 00 for the discharges of 33,500 cu ft per sec and 40,000 cu ft per sec would be shown in Fig. 2. The curves for the other reaches could be shown in a similar manner.

Mr. Ezra uses a weighted value for the velocity of flow when both channel flow and overbank flow are involved. A suggested method is to weight the velocities in accordance with the respective channel and overbank discharges. A weighted-area curve which can be applied directly to the discharges to obtain the weighted velocities can then be derived. Therefore,

$$\bar{V} = \frac{Q_c V_c + Q_o V_o}{Q} \dots \dots \dots (5)$$

in which \bar{V} is the weighted velocity in feet per second, Q_c is the channel flow in cubic feet per second, V_c denotes the velocity of flow in the channel, Q_o represents the overbank flow in cubic feet per second, V_o is the velocity of overbank flow, and Q is the total discharge in cubic feet per second.

Since

$$V = \frac{Q}{A} \dots \dots \dots (6a)$$

then

$$\frac{Q^2}{A_w} = \frac{Q^2_c}{A_c} + \frac{Q^2_o}{A_o} \dots \dots \dots (6b)$$

in which A_w is the weighted cross-sectional area and A_c and A_o are the cross-sectional areas of the channel flow and the overbank flow, respectively.

From the Manning formula,

and

in which

$$C = \frac{1.486}{n} \frac{A R^4}{L^4} \dots \dots \dots \quad (8)$$

and h_f is the friction loss in feet in the distance L . From Eq. 6b,

$$\frac{C^2 h^2_f}{A_v} = \frac{C^2_e h_f}{A_e} + \frac{C^2_o h_f}{A_o} \dots \dots \dots \quad (9a)$$

<or>

$$\frac{1}{A_{\infty}} = \frac{1}{A_c} \left(\frac{C_c}{C} \right)^2 + \frac{1}{A_o} \left(\frac{C_o}{C} \right)^2 \dots \dots \dots \quad (9b)$$

from which

$$A_w = A_e \left[\frac{\left(\frac{C}{C_e} \right)^2}{1 + \left(\frac{A_e}{A_o} \right) \left(\frac{C_o}{C_e} \right)^2} \right] \dots \dots \dots \quad (9c)$$

Either Eq. 9b or Eq. 9c can be used to compute the weighted-area curve. Where the channel and overbank lengths are equal, the term L will cancel and therefore it can be omitted from Eq. 8. Where the reach lengths are different for channel and overbank, the average lengths for adjacent reaches upstream and downstream from the section should be used in Eq. 7.

If the velocity head rather than the velocity is weighted, the expression for the weighted-area curve will be

$$A_w = A_c \left\{ \frac{1}{\left(\frac{C_c}{C} \right)^3 + \left(\frac{C_o}{C} \right)^3 \left(\frac{A_c}{A_o} \right)^2} \right\} \dots \dots \dots (10)$$

A number of graphical and semigraphical charts and graphs have been developed for eliminating the trial-and-error computations in tracing the water-surface profile. The writer devised a method which required a nomograph for the solution.⁵ In most methods the velocity-head corrections have

⁸ "The Nomograph As an Aid in Computing Backwater Curves," by I. H. Steinberg, *Civil Engineering*, Vol. 9, 1939, p. 365.

not been taken into account. However, Francis F. Escoffier has presented a solution which is similar to the direct step method developed by Mr. Ezra in the respect that pairs of curves are plotted for each reach. A sloping straight line drawn from the known point on one curve to the intersection of the second curve gives the required water-surface elevation. The slope of the line is a function of the discharge. Thus, only one set of basic curves is required for a wide range of discharges.

The writer has also developed a graphical solution utilizing "slope curves" in which velocity head corrections are provided for. In this method the water-surface elevations at the lower end of the reach and the discharges are plotted as ordinates and abscissas, respectively, whereas the falls, or differences in water-surface elevation between the upstream and downstream end of the reach, are plotted as the parameter. The curves can also be plotted with the elevation at the upper end of the reach as the parameter. Eq. 1 can be written as

$$Z_1 + \frac{V^2_1}{2g} = Z_2 + \frac{V^2_2}{2g} + h_f + h_o. \dots \dots \dots \quad (11a)$$

in which

$$h_o = K \left(\frac{V^2_1}{2g} - \frac{V^2_2}{2g} \right) \dots \dots \dots \quad (11b)$$

Using the Manning formula for computing the friction loss,

$$h_f = Q^2 \left(\frac{n}{1.486 A_m R_{\text{im}}^4} \right)^2 L = \left(\frac{Q}{C_m} \right)^2 \dots \dots \dots \quad (12a)$$

in which

$$C_m = \frac{1.486}{n} \frac{A_m R^{\frac{1}{m}}}{L^{\frac{1}{m}}} \dots \dots \dots \quad (12b)$$

The subscript m is used to denote the mean value for the reach, and attention is directed to the different method of applying the hydraulic elements to obtain the average slope in the reach from that used in Mr. Ezra's derivation. The mean value for the reach can be based on the average of the end value or on the value as defined by the midpoint elevation of the reach. The writer uses the latter system because of the greater simplicity in computing the basic slope curves.

Eq. 1 can also be written as

$$\text{and } Z_1 - Z_2 = F = \left(\frac{Q_m}{C_m} \right)^2 - \left(\frac{V_{z1}^2}{2g} - \frac{V_{z2}^2}{2g} \right) + K \left(\frac{V_{z1}^2}{2g} - \frac{V_{z2}^2}{2g} \right) \dots (13a)$$

and

$$F = \left(\frac{Q}{C_m} \right)^2 - \left(\frac{V_{2_1}^2}{2g} - \frac{V_{2_2}^2}{2g} \right) (1 - K) \dots \dots \dots (13b)$$

However,

Therefore,

$$F = \left(\frac{Q}{C_m} \right)^2 - \left[\left(\frac{Q}{8.03 A_1} \right)^2 - \left(\frac{Q}{8.03 A_2} \right)^2 \right] (1 - K) \dots \dots (14b)$$

and

$$F = \left(\frac{Q}{C_m} \right)^2 \left\{ 1 - \left[\left(\frac{C_m}{8.03 A_1} \right)^2 - \left(\frac{C_m}{8.03 A_2} \right)^2 \right] (1 - K) \right\} = \left(\frac{Q M}{C_m} \right)^2 \quad (14c)$$

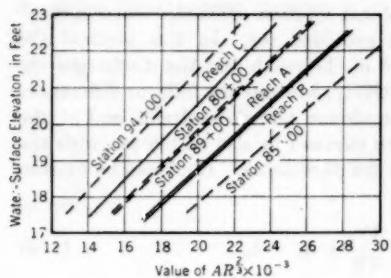
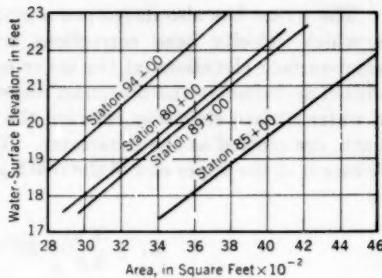
FIG. 4.—ELEVATION—($A R^4$) CURVES

FIG. 5.—ELEVATION—AREA CURVES

in which

$$M^2 = 1 - \left[\left(\frac{C_m}{8.03 A_1} \right)^2 - \left(\frac{C_m}{8.03 A_2} \right)^2 \right] (1 - K) \dots \dots \dots (15a)$$

and

$$Q = \frac{C_m}{M} (F)^{\frac{1}{2}} \dots \dots \dots (15b)$$

Any given values of midpoint elevation and of fall, F , in a reach will define corresponding values of C_m , A_1 , and A_2 . The value of M can be computed from Eq. 15a and the value of Q then determined from Eq. 15b. For development of the "slope curves," Eqs. 15a and 15b are solved for a range of values of midpoint elevation and fall. The resulting points are plotted as curves on appropriate graphs.

To illustrate the application of the "slope curve" graphical method in which a velocity-head correction is included, Example No. 1 will be used. Values of K equal to 0.5 for decelerating flow and K equal to -0.1 for accelerating flow have been adopted. The negative value of K for accelerating flow has been used to conform with Eq. 15a.

Table 7 contains the same basic data as given in Table 1, except that the values of $A R^4$ have been computed. Elevation-($A R^4$) curves and elevation-

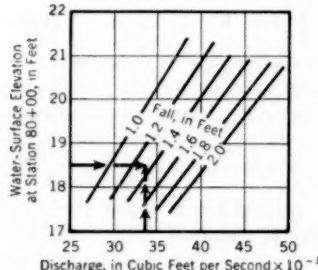


FIG. 6.—SLOPE CURVES FOR REACH A

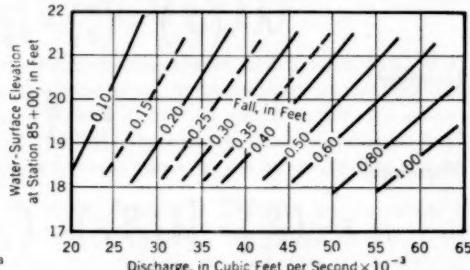


FIG. 7.—SLOPE CURVES FOR REACH B

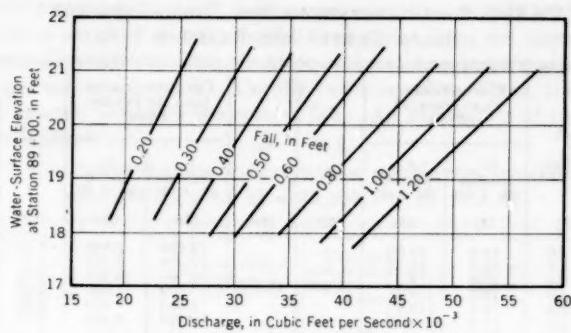


FIG. 8.—SLOPE CURVES FOR REACH C

area curves, based on Table 7, are shown in Figs. 4 and 5. Table 8 illustrates the computations for reach A (station 80 + 00 to station 85 + 00). The values of $A R^{\frac{1}{2}}$ (Col. 5) and areas (Cols. 6 and 7) were obtained from the curves in Figs. 4 and 5. The resulting slope curves are shown in Fig. 6. The curves for reaches B (station 85 + 00 to station 89 + 00) and C (station 89 + 00 to station 94 + 00) are shown in Figs. 7 and 8.

TABLE 7.—HYDRAULIC PROPERTIES OF CROSS SECTIONS

Station (1)	Water-surface elevation, Z , in feet (2)	Cross-sectional area, A , in square feet (3)	Wetted perimeter P , in feet (4)	$A R^{\frac{1}{2}}$ (5)
80 + 00	18.0	3,000	238	16,210
	20.0	3,500	248	20,400
	22.0	3,970	258	24,580
85 + 00	18.0	3,580	265	20,400
	20.0	4,100	273	25,000
89 + 00	22.0	4,630	282	29,800
	18.0	3,080	248	16,400
	20.0	3,550	258	20,500
94 + 00	22.0	4,050	268	24,750
	18.0	2,650	225	13,710
	20.0	3,100	235	17,350
	22.0	3,550	245	21,000

TABLE 8.—COMPUTATIONS FOR DEVELOPING THE
SLOPE CURVES FOR REACH A

Midpoint elevation, in feet	Fall, F , in feet	WATER-SURFACE ELEVATION, IN FEET		$A R^3$	C_m	CROSS-SECTIONAL AREA, IN SQUARE FEET		M^2	Discharge, Q , in cubic feet per second																																												
		Z_2 , Station 80 + 00	Z_1 , Station 85 + 00			A_2 , Station 80 + 00	A_1 , Station 85 + 00																																														
		(1)	(2)	(3)	(4)	(5) ^a	(6) ^b	(7) ^c	(8) ^c	(9) ^d	(10) ^e																																										
18.5	<table border="0"> <tr><td>1.0</td><td>18.0</td><td>19.0</td></tr> <tr><td>1.2</td><td>17.9</td><td>19.1</td></tr> <tr><td>1.4</td><td>17.8</td><td>19.2</td></tr> <tr><td>1.6</td><td>17.7</td><td>19.3</td></tr> <tr><td>1.8</td><td>17.6</td><td>19.4</td></tr> <tr><td>2.0</td><td>17.5</td><td>19.5</td></tr> </table>	1.0	18.0	19.0	1.2	17.9	19.1	1.4	17.8	19.2	1.6	17.7	19.3	1.8	17.6	19.4	2.0	17.5	19.5	19,450	43,100			<table border="0"> <tr><td>3,000</td><td>3,840</td><td>2.39</td><td>27,800</td></tr> <tr><td>2,970</td><td>3,863</td><td>2.47</td><td>30,000</td></tr> <tr><td>2,950</td><td>3,895</td><td>2.55</td><td>31,800</td></tr> <tr><td>2,920</td><td>3,920</td><td>2.63</td><td>33,600</td></tr> <tr><td>2,900</td><td>3,950</td><td>2.71</td><td>35,050</td></tr> <tr><td>2,880</td><td>3,975</td><td>2.80</td><td>36,400</td></tr> </table>						3,000	3,840	2.39	27,800	2,970	3,863	2.47	30,000	2,950	3,895	2.55	31,800	2,920	3,920	2.63	33,600	2,900	3,950	2.71	35,050	2,880	3,975	2.80	36,400
1.0	18.0	19.0																																																			
1.2	17.9	19.1																																																			
1.4	17.8	19.2																																																			
1.6	17.7	19.3																																																			
1.8	17.6	19.4																																																			
2.0	17.5	19.5																																																			
3,000	3,840	2.39	27,800																																																		
2,970	3,863	2.47	30,000																																																		
2,950	3,895	2.55	31,800																																																		
2,920	3,920	2.63	33,600																																																		
2,900	3,950	2.71	35,050																																																		
2,880	3,975	2.80	36,400																																																		
19.5	<table border="0"> <tr><td>1.0</td><td>19.0</td><td>20.0</td></tr> <tr><td>1.2</td><td>18.9</td><td>20.1</td></tr> <tr><td>1.4</td><td>18.8</td><td>20.2</td></tr> <tr><td>1.6</td><td>18.7</td><td>20.3</td></tr> <tr><td>1.8</td><td>18.6</td><td>20.4</td></tr> <tr><td>2.0</td><td>18.5</td><td>20.5</td></tr> </table>	1.0	19.0	20.0	1.2	18.9	20.1	1.4	18.8	20.2	1.6	18.7	20.3	1.8	18.6	20.4	2.0	18.5	20.5	21,600	47,800			<table border="0"> <tr><td>3,255</td><td>4,100</td><td>2.37</td><td>31,050</td></tr> <tr><td>3,230</td><td>4,130</td><td>2.45</td><td>33,500</td></tr> <tr><td>3,205</td><td>4,155</td><td>2.54</td><td>35,600</td></tr> <tr><td>3,180</td><td>4,185</td><td>2.62</td><td>37,400</td></tr> <tr><td>3,155</td><td>4,210</td><td>2.71</td><td>38,900</td></tr> <tr><td>3,130</td><td>4,235</td><td>2.79</td><td>40,500</td></tr> </table>						3,255	4,100	2.37	31,050	3,230	4,130	2.45	33,500	3,205	4,155	2.54	35,600	3,180	4,185	2.62	37,400	3,155	4,210	2.71	38,900	3,130	4,235	2.79	40,500
1.0	19.0	20.0																																																			
1.2	18.9	20.1																																																			
1.4	18.8	20.2																																																			
1.6	18.7	20.3																																																			
1.8	18.6	20.4																																																			
2.0	18.5	20.5																																																			
3,255	4,100	2.37	31,050																																																		
3,230	4,130	2.45	33,500																																																		
3,205	4,155	2.54	35,600																																																		
3,180	4,185	2.62	37,400																																																		
3,155	4,210	2.71	38,900																																																		
3,130	4,235	2.79	40,500																																																		
20.5	<table border="0"> <tr><td>1.0</td><td>20.0</td><td>21.0</td></tr> <tr><td>1.2</td><td>19.9</td><td>21.1</td></tr> <tr><td>1.4</td><td>19.8</td><td>21.2</td></tr> <tr><td>1.6</td><td>19.7</td><td>21.3</td></tr> <tr><td>1.8</td><td>19.6</td><td>21.4</td></tr> <tr><td>2.0</td><td>19.5</td><td>21.5</td></tr> </table>	1.0	20.0	21.0	1.2	19.9	21.1	1.4	19.8	21.2	1.6	19.7	21.3	1.8	19.6	21.4	2.0	19.5	21.5	23,800	52,600			<table border="0"> <tr><td>3,500</td><td>4,370</td><td>2.40</td><td>33,900</td></tr> <tr><td>3,475</td><td>4,395</td><td>2.48</td><td>36,600</td></tr> <tr><td>3,450</td><td>4,420</td><td>2.56</td><td>38,900</td></tr> <tr><td>3,425</td><td>4,445</td><td>2.65</td><td>40,900</td></tr> <tr><td>3,400</td><td>4,470</td><td>2.74</td><td>42,900</td></tr> <tr><td>3,380</td><td>4,500</td><td>2.82</td><td>44,400</td></tr> </table>						3,500	4,370	2.40	33,900	3,475	4,395	2.48	36,600	3,450	4,420	2.56	38,900	3,425	4,445	2.65	40,900	3,400	4,470	2.74	42,900	3,380	4,500	2.82	44,400
1.0	20.0	21.0																																																			
1.2	19.9	21.1																																																			
1.4	19.8	21.2																																																			
1.6	19.7	21.3																																																			
1.8	19.6	21.4																																																			
2.0	19.5	21.5																																																			
3,500	4,370	2.40	33,900																																																		
3,475	4,395	2.48	36,600																																																		
3,450	4,420	2.56	38,900																																																		
3,425	4,445	2.65	40,900																																																		
3,400	4,470	2.74	42,900																																																		
3,380	4,500	2.82	44,400																																																		
21.5	<table border="0"> <tr><td>1.0</td><td>21.0</td><td>22.0</td></tr> <tr><td>1.2</td><td>20.9</td><td>22.1</td></tr> <tr><td>1.4</td><td>20.8</td><td>22.2</td></tr> <tr><td>1.6</td><td>20.7</td><td>22.3</td></tr> <tr><td>1.8</td><td>20.6</td><td>22.4</td></tr> <tr><td>2.0</td><td>20.5</td><td>22.5</td></tr> </table>	1.0	21.0	22.0	1.2	20.9	22.1	1.4	20.8	22.2	1.6	20.7	22.3	1.8	20.6	22.4	2.0	20.5	22.5	26,000	57,500			<table border="0"> <tr><td>3,740</td><td>4,630</td><td>2.41</td><td>37,050</td></tr> <tr><td>3,715</td><td>4,655</td><td>2.49</td><td>39,950</td></tr> <tr><td>3,690</td><td>4,685</td><td>2.58</td><td>42,400</td></tr> <tr><td>3,665</td><td>4,710</td><td>2.67</td><td>44,500</td></tr> <tr><td>3,640</td><td>4,735</td><td>2.75</td><td>46,600</td></tr> <tr><td>3,620</td><td>4,760</td><td>2.83</td><td>48,200</td></tr> </table>						3,740	4,630	2.41	37,050	3,715	4,655	2.49	39,950	3,690	4,685	2.58	42,400	3,665	4,710	2.67	44,500	3,640	4,735	2.75	46,600	3,620	4,760	2.83	48,200
1.0	21.0	22.0																																																			
1.2	20.9	22.1																																																			
1.4	20.8	22.2																																																			
1.6	20.7	22.3																																																			
1.8	20.6	22.4																																																			
2.0	20.5	22.5																																																			
3,740	4,630	2.41	37,050																																																		
3,715	4,655	2.49	39,950																																																		
3,690	4,685	2.58	42,400																																																		
3,665	4,710	2.67	44,500																																																		
3,640	4,735	2.75	46,600																																																		
3,620	4,760	2.83	48,200																																																		

^a Values of $A R^3$ obtained from Fig. 4. ^b Values of C_m determined from Eq. 12b with $n = 0.030$ and $L = 500$ ft. ^c Values of A obtained from Fig. 5. ^d Values of M^2 computed from Eq. 15a with $K = 0.5$ when $A_1 < A_2$ and $K = -0.1$ when $A_1 > A_2$. ^e Values of Q determined from Eq. 15b.

To compute the water-surface profile, it is necessary only (1) to enter the slope curves with the given elevation at the lower end of the reach and the given discharge, (2) to read the corresponding fall, and (3) to add to the given elevation to obtain the elevation at the upper end of the reach, which then becomes the given elevation at the lower end of the next upstream reach. This procedure is illustrated in Table 9. The computed water-surface profile

TABLE 9.—COMPUTATION OF WATER-SURFACE PROFILE FOR A VALUE OF $Q = 33,500$ CU FT PER SEC

Station	Reach	Water-surface elevation, Z , in feet	Fall, F , in feet
(1)	(2)	(3)	(4)
80 + 00	A	18.50	1.33
85 + 00	B	19.83	0.21
89 + 00	C	20.04	0.48
94 + 00		20.52	

shown in Table 9 agrees closely with that shown in Table 6.

It should be noted that the slope curves have been drawn for computing upstream. The same curves can be readily adapted, however, for computing

downstream by trial and error. In this case—(a) the elevation at the lower end of the reach is assumed and (b) the fall is obtained from the slope and added to the assumed elevation. This elevation is checked against the given elevation at the upper end of the reach. Usually two, or perhaps three, trial computations are required to obtain the correct fall. An illustration of the procedure is given in Table 10.

TABLE 10.—DOWNSTREAM COMPUTATION OF WATER-SURFACE PROFILE FOR A VALUE OF $Q = 33,500$ CU FT PER SEC

Station (1)	Reach (2)	Water-surface elevation, Z , in feet (3)	Trial water-surface elevation at lower end of reach (4)	Fall, F , in feet (5)	Computed water-surface elevation at upper end of reach (6)
94 + 00	A	21.00	20.60	0.42	21.02
89 + 00		20.57	20.57	0.43	21.00
85 + 00	B	20.38	20.35	0.19	20.54
80 + 00		19.20	20.38	0.19	20.57
			19.25	1.18	20.43
			19.20	1.18	20.38

C. A. M. GRAY,⁶ A. M. ASCE.—An ingenious variant of the standard step

⁶ Senior Lecturer in Civ. Eng., Univ. of Sydney, Sydney, Australia.

method has been presented by the author. As the solution is direct and the labor involved is considerable, the question of how long the steps are to be made is most important. It seems worthwhile, therefore, to investigate the measure of the error introduced by the selection of an arbitrary length of step.

Eq. 2, neglecting the eddy-loss term, is obtained from the "exact" integral relationship,

$$Z_2 + \frac{V^2_2}{2g} = Z_1 + \frac{V^2_1}{2g} - \int_1^2 S dx \dots \dots \dots \quad (16)$$

when the integral is evaluated by the trapezoidal rule. An alternative value would be $S_M L$ in which S_M is the value of S at the midpoint of the interval L . If the integral is denoted by I , it can be shown that

$$\frac{S_1 + S_2}{2} L > I > S_M L \dots \dots \dots \quad (17a)$$

or

$$S_M L > I > \frac{S_1 + S_2}{2} L \dots \dots \dots \quad (17b)$$

depending on whether the curve of S versus x is concave upward or concave downward. In the former case, Mr. Ezra's solution would give too small a value for Z , and in the latter case it would result in too great a value. Using the mean value of S will give an opposite result. An estimate of S_M can be obtained by use of the data required for Fig. 2. If

$$\left(Z + \frac{V^2}{2g} \right)_{x=L/2} = Z_1 + \frac{V^2_1}{2g} - j \dots \dots \dots \quad (18a)$$

and

$$\lambda < S < \mu \dots \dots \dots \quad (18b)$$

in the range $0 < x < L$, then

$$\frac{\lambda L}{2} < j < \frac{\mu L}{2} \dots \dots \dots \quad (18c)$$

Hence,

$$Z_1 + \frac{V^2 a}{2} - \frac{\mu L}{2} < E_M < Z_1 + \frac{V^2 a}{2} - \frac{\lambda L}{2}. \quad \dots \dots \dots (19a)$$

in which

$$E_M = \left(\frac{V^2}{2g} + Z \right)_{\tau=1/2} \dots \dots \dots \quad (19b)$$

From the properties of the section at $x = L/2$, limits can be set for S_M . Unfortunately, the data are not available. For purposes of investigation it can be assumed, however, that the cross section is constant between stations.

From Tables 1 and 3 it can be seen that, in the interval from station 80 + 00 to station 85 + 00,

$$0.00070 \leq S \leq 0.00162 \quad \dots \dots \dots \quad (20g)$$

and the value of E at station $80 + 00$ is 20.31. Hence, from Eq. 19a,

$$0.40 + 20.31 = 20.71 \geq E_M \geq 0.19 + 20.31 = 20.50, \dots \quad (20b)$$

From Table 1

$$0.00149 \geq S_{\text{eff}} \geq 0.00139 \quad (20c)$$

which is greater than the mean value $\frac{S_1 + S_2}{2}$, indicating that the curve of S versus x is concave downward.

Hence, at station 85 ± 00 ,

$$E \leq 20.31 + 0.75 \equiv 21.06 \quad (21a)$$

and from Table 1

$$Z \leq 20.02 \quad (21b)$$

Under these conditions, at station 85 ± 00 ,

and

Upper and lower bounds to Z at station $85 + 00$ have thus been established, and the separation of these values gives a measure of the accuracy achieved with the assumed length of step.

BOB BUEHLER,⁷ A. M. ASCE.—A method has been presented by Mr. Ezra
⁷ Hydr. Engr., TVA, Knoxville, Tenn.

which should be welcomed by those engaged in computing water-surface profiles in natural streams or reservoirs. If profiles for several flows and downstream starting elevations are to be determined, use of this method, as opposed to the standard trial-and-error procedure, will save time and avoid drudgery. The method has been tried successfully on a typical stream, and it is believed to be superior to most others.

This discussion (1) seeks confirmation or simplification of a method presented by the writer for the consideration of overbank flow; (2) offers alternate plotting procedures; and (3) seeks further confirmation of (a) the application of channel-roughness coefficients which vary from reach to reach, (b) the determination of these coefficients from observed flood profiles, and (c) the determination of channel coefficients under conditions in which there is a substantial amount of overbank flow.

Inclusion of Overbank Flow.—Table 11, showing hydraulic properties, is a

TABLE 11.—HYDRAULIC PROPERTIES OF CROSS SECTIONS FOR $Q = 33,500$ CU FT PER SEC, n FOR CHANNEL = 0.03, AND n FOR OVERBANK = 0.05

Station (1)	Water-surface elevation Z , in feet (2)	Cross-sectional area A , in square feet (3)	Hydraulic radius R , in feet (4)	$Q \cdot S^{-\frac{1}{2}}$ (5)	Flow Q , in cubic feet per second (6)	$S^{\frac{1}{2}}$ (7)	Friction gradient (8)	Velocity V , in feet per second (9)	$V^2 Q$ (10)	Weighted V^2 (11)	$\frac{V^2}{Q}$ (12)
80 + 00	18.0	3,000 0	12.60 0.00	805,000 0	33,500	0.0416	0.00173	11.17			1.94
	20.0	3,500 500	14.11 3.00	1,012,000 30,900	32,508 992			9.29 1.98	2,800,000 3,890		
				1,042,900	33,500	0.0321	0.00103		2,803,890	83.7	1.30
	22.0	3,970 1,000	15.39 6.00	1,217,000 98,100	31,000 2,500			7.81 2.50	1,891,000 15,620		
				1,315,100	33,500	0.0255	0.000650		1,906,620	56.9	0.88
	18.0	3,580 0	13.51 0.00	1,005,000 0	33,500	0.0333	0.00111	9.36			1.36
85 + 00	20.0	4,100 300	15.02 1.70	1,237,000 12,660	33,161 339			8.09 1.13	2,170,000 433		
				1,249,660	33,500	0.0268	0.000718		2,170,433	64.8	1.01
	22.0	4,630 900	16.42 5.00	1,482,000 78,100	31,823 1,677			6.87 1.86	1,502,000 5,800		
				1,560,100	33,500	0.0215	0.000462		1,507,800	45.0	0.70

variation of Table 1 in that flow in an assumed overbank has been added. For brevity, only the reach from station 80 + 00 to station 85 + 00 has been used. Table 11 shows, for each elevation, two values for the area and two values for the hydraulic radius. The upper figure is the channel value used in Table 1, and the lower figure is the overbank value that has been assumed for illustration. The value of n for the overbank has been taken as 0.05. If discharge is represented by

$$Q = \frac{1.486 A R^{\frac{1}{2}} S^{\frac{1}{2}}}{n} \dots \dots \dots (23a)$$

then

$$\frac{Q}{S^{\frac{1}{2}}} = \frac{1.486 A R^{\frac{1}{2}}}{n} \dots \dots \dots (23b)$$

In Col. 5, Table 11, $Q S^{-1}$ can be computed first for the channel section, and next for the overbank; then the two values are added. For El. 20.0 at station 80 + 00, $Q S^{-1}$ is 1,012,000 in the channel, 30,900 in the overbank, thus totalling 1,042,900. The true total discharge is known to be 33,500 cu ft per sec. Therefore, Cols. 7 and 8 (Table 11) can be determined. With S^1 known, the parts of the total flow carried respectively by the channel and overbank can be computed and entered in Col. 6, Table 11. The velocities and weighted velocity head for the total section can be found and entered in Cols. 9 through 12, Table 11.

Table 12 shows, for this same reach, the computation for $\phi(Z)$ and $f(Z)$ for

TABLE 12.—COMPUTATION FOR $Z + f(Z)$ AND $Z + \phi(Z)$ FOR REACH
FROM STATION 80 + 00 TO STATION 85 + 00 WITH n FOR
CHANNEL = 0.03, n FOR OVERBANK = 0.05,
AND $L/2 = 250$ FT

Station	Water-surface elevation Z , in ft	VALUES FOR $Q = 33,500$ CU FT PER SEC				VALUES OF $\phi(Z)$ OR $f(Z)$ FOR FLOW Q , IN CU FT PER SEC				VALUES OF $Z + \phi(Z)$ OR $Z + f(Z)$ FOR FLOW Q , IN CU FT PER SEC				
		S	$\frac{L}{2}$	Weighted $\frac{V^2}{2g}$	$\phi(Z)$	$f(Z)$	10,000	20,000	30,000	40,000	10,000	20,000	30,000	40,000
80+00	18.0	0.43	1.94	2.37			0.21	0.84	1.90	3.37	18.21	18.81	19.90	21.37
	20.0	0.26	1.30	1.56			0.14	0.56	1.25	2.22	20.14	20.56	21.25	22.22
	22.0	0.16	0.88	1.04			0.09	0.37	0.83	1.48	22.09	22.37	22.83	23.48
85+00	18.0	0.28	1.36		1.08	0.10	0.38	0.87	1.54	18.10	18.38	18.87	19.54	
	20.0	0.18	1.01		0.83	0.07	0.30	0.66	1.18	20.07	20.30	20.66	21.18	
	22.0	0.12	0.70		0.58	0.05	0.21	0.46	0.83	22.05	22.21	22.46	22.83	

the base flow of 33,500 cu ft per sec using the slopes and weighted velocity heads found in Table 11. Other entries in Table 12 will be explained subsequently.

Alternate Plotting Procedures.—Adaptation to specific needs as well as to individual preference determines the most convenient method of plotting. For example, if curves were to be prepared which would be suited to a wide variety of flows, the form used by the author would become unwieldy. It would seem logical, therefore, to use one plotting sheet for each reach, showing on each a family of discharge curves. An example for the reach from station 80 + 00 to station 85 + 00 is shown in Fig. 9 which was prepared from Table 12. Values of $\phi(Z)$ and $f(Z)$ in Table 12 for the various flows were computed from the base value for 33,500 cu ft per sec, using the ratio of the discharges squared, as suggested by Mr. Ezra. The solid lines in Fig. 9 were drawn from computed values of $Z + \phi(Z)$ and $Z + f(Z)$. The dashed intermediate lines were interpolated by use of construction lines based on the computed curves. The form of this plot differs from that presented by the author. In this form, if eddy losses are ignored, the upper curves can be entered with the known elevation at the downstream section and flow—for example, El. 20.32 at station 80 + 00 and 37,000 cu ft per sec. This intersection is carried vertically downward to the lower curves until the proper discharge is reached. This lower intersection

gives a resulting elevation of 21.28 at station 85 + 00. The values of $Z + \phi(Z)$ and $Z + f(Z)$ need not be tabulated since they only represent guide lines connecting the upper curves for station 80 + 00 to the lower curves for station 85 + 00.

The basic form of the curve in which Z is plotted against $Z + \phi(Z)$ and $Z + f(Z)$ has serious limitations if the water-surface elevations at a given section will vary over a wide range. A scale on a convenient size of graph paper can be read with sufficient accuracy if the graph covers an elevation range of perhaps 10 ft, but not if it covers a range of 50 ft or more. For any given reach, however, there usually is a reasonable limit in the elevation differences at the two ends. The functions $\phi(Z)$ and $f(Z)$ are closely related to the rise in a reach, suggesting a rearrangement of Eq. 4. Neglecting eddy losses, Eq. 4 can be written as

$$Z_1 = Z_2 + \phi(Z_2) - f(Z_1) \dots \dots \dots \quad (24)$$

For any reach, a curve can be drawn for the downstream section showing $\phi(Z)$ versus Z . Another curve can also be drawn for the upstream section showing $f(Z)$ against $Z + f(Z)$. Fig. 10 is an example of this in which the reach from station 80 + 00 to 85 + 00 has been used. Fig. 10 was plotted from the data in Table 12. Table 13 shows the water-surface profile for a flow

TABLE 13.—WATER-SURFACE
PROFILE FOR $Q = 37,000$
CU FT PER SEC

Station (1)	Water-surface elevation Z , in feet (2)	Value of $\phi(Z)$ (3)	Value of $Z + \phi(Z)$ (4)	Value of $f(Z)$ (5)
80 + 00	20.32	1.78	22.10	0.82
85 + 00	21.28			

of 37,000 cu ft per sec and illustrates the use of the curves. Starting with El. 20.32 or Z at station 85 + 00, $\phi(Z)$ (Col. 3, Table 12) is found to be 1.78 from the upper curves. This value is added to Z to get a value of $Z + \phi(Z)$ equal to 22.10 (Col. 4, Table 13). From Eq. 4, however, this value equals $Z + f(Z)$ at station 85 + 00. Entering with this value of 22.10 in the lower curves, $f(Z_1)$ (Col. 5, Table 13), is found to be 0.82. The value of Z at station 85 + 00 (El. 21.28) is Col. 5, Table 13, subtracted from Col. 4, Table 13. In this procedure, elevations are not read from the curves. Instead, factors comprising the difference in elevations are read. The operation requires more tabulating than when Fig. 9 is used; but with the same size graph paper, it is far more accurate whenever wide elevation ranges must be accommodated. Of course, it is presumed that reaches will be short enough so that values of $\phi(Z)$ and $f(Z)$ will be sufficiently small to plot with reasonable accuracy.

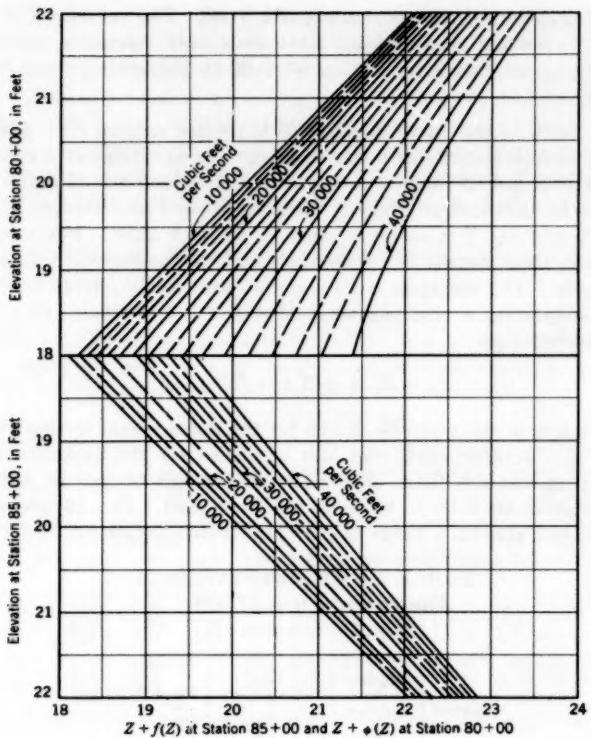


FIG. 9.—CURVES FOR COMPUTING WATER-SURFACE PROFILE

Roughness Coefficients and their Determination.—Whenever possible, it has been the writer's practice to use roughness coefficients that were computed from observed flood profiles for known flows. If a choice of floods for this purpose is available, the preferred flood is one which just fills the channel part of the cross section without extending into the overbank. The resulting computed channel coefficients are then tested on one or more higher floods to aid in assigning a suitable roughness coefficient to the overbank. Channel-roughness values usually vary enough, and are critical enough, so that different values are required in each reach. Overbank values are (a) more difficult to compute with assurance, (b) are usually less critical than channel values, and (c) are often assumed to be constant for long stretches of a stream. Channel-roughness factors are computed, and applied, with the friction slope based on the average of the areas and the hydraulic radii at the two sections bounding each reach.

Mr. Ezra uses a different procedure in the application of area and of hydraulic radius. The rate of friction slope is found for each section using its area and hydraulic radius. The total friction slope in a reach is, then, the average of the rates at its two end points times the length. If roughness factors vary in each

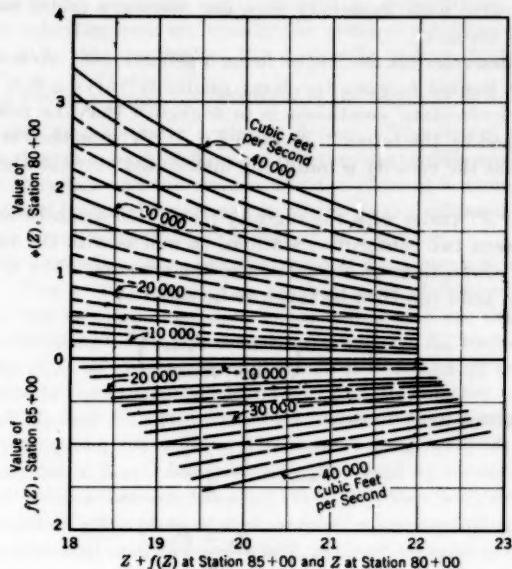


FIG. 10.—CURVES OF $\phi(Z)$ AND $f(Z)$ FOR COMPUTING WATER-SURFACE PROFILES

reach, the rate of friction loss at each section must be computed twice—once for use in an upstream direction, and once for use in a downstream direction. This would require two columns for S in Table 1, one headed S_u and one S_d . It would be similar to the need for both Cols. 3 and 4, in Table 2, in which $\frac{1}{2} L$ may differ in the upstream and downstream directions. Also, Col. 3, Table 2, would then be entitled $\frac{1}{2} S_d L_d$ and Col. 4, Table 2, would be entitled $\frac{1}{2} S_u L_u$.

When overbank flow is not involved, roughness factors can presumably be computed from a marked flood profile of known flow using Eq. 3. All quantities in this equation are known except the value of n . When n must be computed from a flood involving appreciable overbank flow, however, the determination of the channel values, even with the value of n in the overbanks assumed, becomes a cumbersome operation. Were it not necessary to weight the channel and overbank velocity heads, the overbank flow could be computed first and deducted from the known total flow, thereby leaving the necessary known quantities with which the channel roughness can be computed. This would be a reasonable expedient if the overbank flows were not large. When overflow is large, however, and when it appears necessary to use a weighted velocity head (even with an assumed overbank n), it seems necessary to make successive estimates of the value of n for the channel until a value is found that will satisfy Eq. 3 (modified to include overbank flow). An application of this principle to a typical stream required the use of a 20-column table to reach a satisfactory solution.

MONIR M. KANSOH.⁸—The method proposed by the author is a valuable

⁸ Asst. Prof., Faculty of Eng., Univ. of Alexandria, Alexandria, Egypt.

contribution to the problem of plotting water-surface profiles. It facilitates

plotting and saves time, especially once the necessary tables and graphs are computed and prepared.

Mr. Ezra describes his method as being a general one. Actually, however, the method is limited to cases involving gradually varying flow in which departure from hydrostatic conditions is so negligible that the potential energy can be expressed by the terms Z_1 or Z_2 in Eq. 1. The method is based on the assumption that the velocity is uniformly distributed over the area of any cross section.

The term "S" varies with the rugosity factor which can be supposed to be constant between two consecutive stations, as well as with the velocity of flow and with the proportions of the section of the channel. The average rate of friction loss of head can then be taken to be equal to

$$S_{\text{aver}} = \left(\frac{n}{1.486} \frac{V_{\text{aver}}}{R_{\text{aver}}^{2/3}} \right)^2 \dots \dots \dots \quad (25)$$

Assuming, in a uniform canal, that

$$V_{\text{aver}} = \frac{V_1 + V_2}{2} \dots \dots \dots \quad (26a)$$

and

$$R_{\text{aver}} = \frac{R_1 + R_2}{2} \dots \dots \dots \quad (26b)$$

then

$$\begin{aligned} S_{\text{aver}} &= \left(\frac{n}{1.486} \right)^2 \left(\frac{V_1 + V_2}{2} \right)^2 \left(\frac{2}{R_1 + R_2} \right)^{4/3} \\ &= \left(\frac{1}{2} \right)^{2/3} \frac{n^2}{2.2} \frac{(V_1 + V_2)^2}{(R_1 + R_2)^{4/3}} \end{aligned} \quad \dots \dots \dots \quad (27)$$

Eq. 27 shows that S_{aver} , as derived, does not equal $\frac{1}{2} (S_1 + S_2)$, nor $\frac{1}{2} \frac{n^2}{2.2} \left(\frac{V_{21}^2}{R_{1/3}^2} + \frac{V_{22}^2}{R_{2/3}^2} \right)$ as proposed by the author.

It should be realized that the area, the perimeter, the hydraulic mean depth or radius, and consequently $\frac{Q^2}{g A^2}$ and $\frac{Q^2 n^2}{2.2 A^2 R^{4/3}}$ are functions of the depth of flow and not of the elevation of the water surface above a chosen datum. It is not correct, then, to denote that (in general)

$$\frac{Q^2}{2 g A^2} \pm \frac{L}{2} \frac{Q^2 n^2}{2.2 A^2 R^{4/3}} = \phi(Z) \text{ and } f(Z).$$

This is true only in particular cases for particular cross sections selected beforehand.

It would be helpful if a plot were presented of the same surface profiles computed by Mr. Ezra's method and according to the methods developed by Bresse, Bakhmeteff, and Tolkmitt (as well as some of the simplified analytical and graphical methods) to show the discrepancy.

ARTHUR A. EZRA,⁹ J. M. ASCE.—Mr. Kansoh is correct in stating that the

⁹ Senior Harbor Engr., Port of San Francisco, Calif.

proposed method is limited to conditions in which the potential energy can be expressed by the terms Z_1 or Z_2 in Eq. 1. This limitation corresponds to the boundary conditions (found in the theory of varied flow in open channels) on

which are based the generally used methods of determining water-surface profiles. The velocities used are usually the mean or weighted velocities at a section. The area, the perimeter, the hydraulic radius, and consequently the terms $\frac{A^2}{g A^2}$ and $\frac{A^2 n^2}{2.2 A^2 R^{4/3}}$ are functions of the depth of flow and the elevation of the water surface above a chosen datum. The reason for this interrelationship is that the depth of flow is a function of the difference between the water-surface elevation and the elevation of the channel bottom. The elevation of the channel bottom is expressed implicitly in the values of A and R that correspond to a given water-surface elevation at a section.

Mr. Kansoh's question regarding the value of S_{aver} appears to be a matter of definition. The writer defined S_{aver} as that value of S which is the arithmetic mean of the values S_1 and S_2 . These values of S are determined individually from Manning's formula using V_1 , R_1 , and V_2 , R_2 , respectively. Mr. Kansoh defines S_{aver} as that value of S which is obtained by substituting in Manning's formula the arithmetic mean of V_1 and V_2 together with the arithmetic mean of R_1 and R_2 . Naturally, the value of S_{aver} as defined by Mr. Kansoh will not equal $\frac{1}{2}(S_1 + S_2)$. The writer's usage of $\frac{1}{2}(S_1 + S_2)$ as the average rate of loss of head corresponds to that used by George E. Russell.¹⁰

¹⁰ "Hydraulics," by George E. Russell, Henry Holt and Co., New York, N. Y., 5th Ed., 1947, p. 292. The writer's method gives plots of surface profiles corresponding to those obtained by the standard step method which is based on trial and error. Mr. Kansoh's suggestion of plotting the same surface profiles for comparison by Bresse's, Bakhmeteff's, and Tolkmitt's methods is profitable.

The writer is indebted to Mr. Gray for his analysis of the relative accuracy of the approximation used to establish the friction-head loss in a reach in comparison to the results obtained by using $\int S dx$ and $S_M L$. From Table 3 the difference in water-surface elevations between station 85 + 00 and station 80 + 00 is 1.33 ft. Using Mr. Gray's method of analysis, the difference in water-surface elevations computed by using $\int_1^2 S dx$ is between 1.33 ft and 1.52 ft. On the basis of this analysis, the writer's approximation yields a result for a 500-ft-long interval that is within 12% of the result that would be obtained by using $\int S dx$. This inaccuracy is small in comparison to the inaccuracies involved in the choice of a value of the roughness coefficient n , especially since S varies as n^2 .

Mr. Buehler has made a valuable contribution to the method of computing water-surface profiles. His suggestions for alternate plotting procedures are very appropriate. It is agreed that one plotting sheet should be used for each reach, as shown in Fig. 9. The form used in Fig. 2 was presented for the purpose of illustrating the application of the proposed method. The method of plotting shown in Fig. 10 is an ingenious solution to the practical difficulty involved in plotting a graph to cover a range of 50 ft in elevation at a section. It has been found that the method of inclusion of overbank flow, proposed in Eqs. 23 and Table 11, is a commonly used expedient. Basically, this method corresponds to the use of the function,¹¹

¹¹ "Handbook of Hydraulics," by H. W. King, McGraw-Hill Book Co., Inc., New York, N. Y., 1939, pp. 530-531.

Although some engineers may object to this procedure on the grounds that the value of $A R^1$ for a cross section taken as a whole is not equal to the sum of the $(A R^1)$ -values of its component parts, it should be realized that the difference is not great for moderate amounts of overbank flow. If the value of n is determined from observed flood profiles (using the sum of the $(A R^1)$ -values for channel and overbank), the computation of surface profiles should also use this n -value.

Mr. Buehler is correct in his interpretation of the procedure to be followed in applying the proposed method to channels where it is necessary to use a different roughness coefficient for each reach. The computations in Table 11 and Eq. 23 show how it is possible to use different roughness coefficients at the same cross section for different parts of the cross section. The writer agrees with Mr. Buehler that it is a cumbersome operation to determine the value of the roughness factor from flood observations when there is appreciable overbank flow.

Mr. Steinberg makes a welcome suggestion regarding the use of a weighted-area curve that can be applied directly to the discharges to obtain weighted velocities when overbank flow is involved. It has been observed, however, that the use of Eq. 5 involves less work and is simpler than the use of Eq. 9c or Eq. 10 to prepare a weighted-area curve.

Mr. Steinberg's discussion makes it obvious that it was not made sufficiently clear that the proposed method is a rigorous, although semigraphical, solution of the basic equation of varied flow given by Eq. 1. With this solution, it is not necessary to ignore the velocity-head terms or to make trial-and-error computations. The method, although perfectly applicable to channels of uniform cross section and roughness, is especially suited to natural channels in which (1) the velocity varies from reach to reach, (2) consecutive lengths of reach are not equal, (3) each reach has a different roughness coefficient, (4) conditions of overbank flow prevail, (5) the same cross section has different roughness coefficients for different water-surface elevations, and (6) the effect of eddy losses and bridge-pier losses can readily be included when necessary. Mr. Steinberg is correct in showing that, in most graphical and semigraphical methods which seek to eliminate trial-and-error computations, the velocity head corrections have not been taken into account. That is, these methods solve Eq. 1 by

omitting consideration of the terms $\frac{V^2_1}{2g}$ and $\frac{V^2_2}{2g}$, and furthermore by assuming that the differences in elevations Z_1 and Z_2 at the two ends of a reach are equal to

that the difference in elevations Z_1 and Z_2 at the two ends of a reach are equal to the friction-head loss only. Although these simplifying assumptions are useful and not inaccurate when the velocity heads at each end of a reach are equal (in which case there is no varied flow), or when the difference between velocity heads is small compared to the friction-head loss in the reach (which is not the case when short lengths of reach are used under conditions of varied flow), they cannot be termed a rigorous attempt to solve the equation of varied flow.

Mr. Steinberg presents a method of using a graphical solution for the equation of varied flow. Since the computation of M from Eq. 15a requires the determination of the properties of a midsection, as well as of the ends of a reach, the additional work involved is not worthwhile since no greater precision is obtained as Table 9 shows in comparison with Table 6.

**DISCUSSION OF DAM MODIFICATIONS
CHECKED BY HYDRAULIC MODELS
PROCEEDINGS-SEPARATE 184**

G. H. HICKOX,⁷ M. ASCE.—A praiseworthy description of an unusually

⁷ Associate Director, Eng. Experiment Station, Univ. of Tennessee, Knoxville, Tenn.

complete set of model tests has been presented by the authors. This paper presents an excellent example of the type of model testing that is necessary for the design of a new dam or the reconstruction of an old dam. The authors have arrived at completely satisfactory conclusions. Only a few comments, accordingly, on the general subject of model tests for dams shall be added.

It has been assumed that considerable erosion would occur in the river bed below a dam, even in rock, if special provisions were not made to prevent it. There is considerable evidence to support this assumption. Wilson Dam on the Tennessee River, near Florence, Ala., is founded on limestone but it was not provided with an apron. Considerable erosion occurred at the toe when the dam was first placed in operation. Extension of the apron served merely to transfer the erosion to a point further downstream. It seems probable that properly conducted model tests might have abolished this problem. Messrs. Harrison and Kindsvater describe a dam that is twenty-five years old, which, although lacking an apron, has sustained considerable flow without any significant erosion of the river bed. The experiments were conducted on models having erodible beds in which considerable movement occurred. The results of the model tests were used for comparative purposes only, however, and no attempt was made to claim that erosion in the prototype would agree quantitatively with erosion in the model. The fundamental difficulty appears to be a lack of knowledge about the erodibility of various rocks under the flow conditions to which they are subjected. Without such knowledge, the designer must be cautious.

The authors made tests on models at two different scales for various purposes. These objectives were to study the following: (1) Effect of the buttresses on erosive tendencies; (2) effect of sills at the end of the apron on erosion; (3) most suitable schedule of gate operation; (4) location of struts between buttresses; (5) elevation of the powerhouse training wall; (6) elevation and location of construction cofferdams; (7) pressures on the face of the spillway; (8) water surface profiles; and (9) spillway discharge calibration.

A large model of a section of the spillway was built at a 1/40-scale to determine the last three objectives. A smaller, 1/80-scale model of the entire dam, powerhouse, and a part of the river channel was built for the first six objectives. The paper indicates clearly that it is not sufficient to build a large-scale model for one or two bays of a spillway to study the erosion at the toe of a dam. There is usually a problem of erosion at the end of the spillway adjacent to the nonoverflow section of the dam or adjacent to the powerhouse. When-

ever the overflow section of the spillway is away from either bank, an eddy current is set up in the region of relatively quiet water between the end of the spillway and the adjacent river bank. At the base of the dam, or in the powerhouse tailrace, the eddy current moves toward the spillway section where the water surface is depressed. At the junction of the high velocity stream from the spillway and the lateral current from the eddy, local eddy currents of considerable intensity and erosive power are formed. These local eddies cause scouring at the end of the spillway or at the training wall (if a training wall exists at that point). Extension of the training wall in a downstream direction does not eliminate the difficulty except at a prohibitive cost. The solution to the problem usually requires a training wall with a proper combination of height, length, and location, and river bed paving. The proper combination can be determined only by a study of a complete model of the entire spillway and river and cannot be determined by tests made on a sectional model.

The Bartlett's Ferry Dam is unusual with respect to the relative locations of the powerhouse and the spillway. At this dam the powerhouse tailrace is directly below a part of the spillway. The training wall dividing the spillway from the tailrace is subjected to an increase in both water surface and water pressure on the spillway side, rather than reduction in elevation, as is usually the case when the dam is straight.

The results reported as having occurred with the use of a sloping sill at the end of the apron are in agreement with the experience of the Tennessee Valley Authority (TVA). Aprons of most TVA dams are terminated with a sloping sill which reduces the erosive tendencies at the end of the apron. TVA experience also agrees with that reported by Messrs. Harrison and Kindsvater that the slope of the upstream face is usually a compromise between the depth of erosion and the height of the waves produced in the river channel below the dam. The steeper sills produce the least erosion and the highest waves. The flat sills increase erosive tendencies but reduce the wave height. The slope adopted is generally chosen with regard to the importance of erosion at the toe of the sill and the effect of waves on navigation and erosion of the bank.

It is encouraging to note that the authors considered conditions during construction. Model tests of high dams are frequently made without regard to conditions existing during diversion. A stilling basin that may be entirely satisfactory for the maximum flood discharge of a stream after the completion of the dam may be wholly inadequate when it is required to handle a much smaller flood under entirely different conditions. During the construction period, flow may be diverted through a narrowed river channel or, occasionally, through tunnels bypassing a cofferdam. In either case, the flow is concentrated, and considerable damage may result. This is especially true when diversion occurs through spillway blocks which have not reached their final height. This circumstance causes concentration and direction of flow that are quite different from those of the completed structure, and the erosive effects may be quite unexpected. The concentration of flow over a fraction of a long apron may result in eddies that move debris and gravel onto the apron and, by continual movement of this material, cause serious scour. Such conditions can be predicted only by simulating construction conditions and expected floods with models.

It is gratifying to note that observations on the model have been made for rather small prototype discharges. One of the reasons why satisfactory comparisons of model and prototype operations are uncommon is that many models are tested only for the worst design condition, which is usually at or near maximum discharge. Extreme discharges occur very rarely in the prototype. Comparisons of model and prototype are therefore available only when the experimenter has had sufficient foresight to conduct tests in the normal operating range. The meager stock of knowledge about the reliability of models will increase if careful test data are obtained in the range of normal, or relatively frequent, prototype operating conditions.

Another somewhat neglected phase of model testing is the effect of gate operations on erosive tendencies. As indicated by the authors, concentration of flow is usually undesirable and nearly uniform distribution is greatly preferred. Tests on a model with an erodible bed are an extremely valuable method of determining the erosive effect caused by various methods of gate operations. Similar tests with a fixed-bed model are also valuable in determining the effect of concentrated discharges on surface currents which may hinder or improve navigation. It would seem preferable to learn these combinations by model testing rather than by interfering with navigation.

The importance of having model tests designed, carried out, and interpreted by a competent, well-trained staff should be emphasized. It is known that in any argument the conclusions reached are always a function of the initial premises. If the model experimenter assumes that only a single sectional model will be sufficient for the problem at hand, the answers obtained will be a function of the particular model. For example, such a model would be adequate for determining the effect of the height of the sill on erosion. If, however, it is assumed that there may be lateral flow problems, a complete model should be built to explore the situation. In such a case, the results discovered may be other than those predicted by use of the sectional model. Similar considerations apply to the use of fixed and movable beds. Model tests should therefore be made and interpreted by the most competent investigators available. The requirements should include both field experience with operating structures and training in fundamental hydraulics and fluid mechanics.

E. S. HARRISON,⁸ AND CARL E. KINDSVATER,⁹ MEMBERS, ASCE.—The

⁸Asst. Chf. Engr., Georgia Power Co., Atlanta, Ga.

⁹Prof. of Civ. Eng., Georgia Inst. of Technology, Atlanta, Ga.

writers are indebted to Mr. Hickox for his comprehensive discussion of the model testing aspects of the paper. His general comments and admonitions are an invaluable complement to the paper. The writers also wish to acknowledge their use of the uplift measurements made at Bartlett's Ferry Dam¹⁰

¹⁰"Uplift in Masonry Dams": Final Report of the Subcommittee on Uplift in Masonry Dams of the Committee on Masonry Dams of the Power Division, 1951, *Transactions, ASCE*, Vol. 117, 1952, p. 1243. (Georgia).



**DISCUSSION OF SETTLING RATES OF SUSPENSIONS
IN SOLIDS CONTACT UNITS
PROCEEDINGS-SEPARATE 186**

NORVAL E. ANDERSON,⁶ M. ASCE.—The generally accepted theory of
⁶Engr. of Treatment Plant Design, The Sanitary District of Chicago, Chicago, Ill.

quiescent settling has been ably presented by Mr. Kalinske. The author also has presented mathematical functions for some of the variables affecting sedimentation (including upward flow) and has given some practical suggestions for good design (based on empirical results). However, he has not thoroughly presented general information on the settling characteristics of various suspensions, nor has he completely shown how to apply this data to the design and operation of water and waste treating and clarification basins. Data available for use in the equations are few.

If, for example, a designer decided to use Eq. 10 to determine the settling area of an upflow basin, the only known factor would be Q , the inflow. The designer could assume C_r , the solids concentration in the inflow, at 2,000 ppm, and C_s , the solids concentration of the slurry, at 10,000 ppm. This makes the ratio C_r/C_s equal to 1/5. The subsidence rate V or V_s must then be determined. If samples of the suspension were available to make laboratory determinations of the rate of subsidence, there would still remain the question as to how the laboratory rate would be affected in the plant-size unit by stray currents, and how much of a factor should be allowed for bulking or variations in the behavior of the suspension. Hence, the designer may assume a surface rate of 1.5 gal per min per sq ft, or some other empirical figure based on experience with similar suspensions.

Data on the size, shape, density, and other characteristics of the mass of heterogeneous particles encountered in water and waste treatment are comparatively meager. It is questionable whether there will ever be sufficient data of this kind to make a rigorous theoretical analysis of the design of a settling tank for a sewage plant based on the internal mechanics of suspensions. This is because the suspended particles in a single sample are so variable that they defy practical classification. The settling characteristics of these particles change greatly (sometimes within a few hours) in the same plant, and there are often considerable differences in the settling characteristics in different plants handling the same type of wastes. This does not mean that the theoretical approach should be neglected. Knowledge of the mathematical relations, even though they cannot be evaluated numerically, provides a better understanding of empirical results, and sometimes lends new significance to old experimental data.

For example, W. Rudolphs and I. O. Lacy found that activated sludge settled more rapidly in a funnel than in a cylinder, and more rapidly in a 1-liter cylinder when set at an angle than when set vertical.⁷ This could be explained

⁷ "Settling and Compacting of Activated Sludge," by W. Rudolphs and I. O. Lacy, *Sewage Works Journal*, July, 1934, p. 647.

by Eq. 8 and by the statement that

"* * * In an expanding cone the faster settling particles will also remain towards the bottom but, because V' and v both decrease upwards, this will tend to cause the ratio V'/v to change more gradually. Thus the suspension will tend to be of a somewhat more uniform concentration in a cone than in a cylinder."

Thus, if the concentration tends to remain more uniform, the heavier particles will continue to influence the settling rate of the lighter particles for a longer time, resulting in more rapid settling of the sludge mixture; or the observed differences in settling time could have been caused by the differences in the height of the vessels used in the test.

Another example comes from the investigations of the behavior of the final settling tanks at the Southwest Sewage Treatment Works of the Sanitary District of Chicago, Ill., results of which were presented by the writer in 1944.⁸ After it was discovered that density currents were the cause of floc

⁸ "Design of Final Settling Tanks for Activated Sludge," by Norval E. Anderson, *Sewage Works Journal*, January, 1945, p. 50.

passing over the effluent weirs on the periphery of the 126-ft-diameter tanks, data from sludge-density determinations made in 1931 at the Des Plaines River plant in Illinois developed a new relation between the location of the effluent weirs and the sludge draw-off.

With our present state of knowledge (1953), the approach to the design of settling basins through applied mechanics is of great value to the investigator who interprets results from operating units with a view toward improvements. The designer should be familiar with these fundamental relationships to apply intelligently the mass of empirical data which is available. A comprehensive discussion of the settling theory has been presented by Thomas R. Camp,⁵ M. ASCE.

⁵ "Sedimentation and Design of Settling Tanks," by Thomas R. Camp, *Transactions, ASCE*, Vol. 111, 1946, p. 895.

Mr. Kalinske is to be commended for his presentation of theoretical relationships, for emphasizing their importance, and for his suggestions for practical application.

Ross E. McKinney,⁹ J. M. ASCE.—In an effort to present a compact

⁹ Asst. Prof. of San. Eng., Dept. of Civ. and San. Eng., Massachusetts Inst. of Technology, Cambridge, Mass.

description of the action of the solids contact units, the author has tended to oversimplify the conditions encountered in such units. The descriptions of plain settling and upflow suspension were based on the results obtained with suspensions of sand, and it was concluded that the data could be applied to any type of suspension. Sand grains are dense, discrete particles which do not affect one another except by physical packing. The grains retain their identity as individual particles in concentrated suspensions (as they also do in dilute suspensions) and are unaffected by turbulence or any shearing action which

might be created by the velocity of water passing through the suspension. Unfortunately, sand does not represent the type of solids normally removed by solids contact units.

In the clarification of water, the solids contact units are used to remove finely divided suspended matter and colloidal matter with the aid of chemical coagulation. The floc that is formed is not a tough, discrete particle, such as a grain of sand, but is a fragile particle easily affected by other particles and by liquid motion. Chemical floc or biological floc produced during the clarification of sewage is of the same nature. In solids contact units, the upflow rate is not only a factor of the settling velocity of the particle but also a factor of the structural strength of the floc.

At low rates of flow, the quantity of floc formed is small and the upflow velocity is low. Thus the denser flocs will settle readily, leaving a small concentration of light floc in suspension. As the flow increases, the quantity of floc and the upflow velocity also increase. The heavier floc is retained in suspension, and—since more floc is present—the number of contacts with other floc particles increases. If the upflow velocity is less than the shearing velocity of the floc, larger particles will form. The larger particles (increasing in mass and size) will oppose the upflow velocity and tend to become more compact. The suspended floc will form a "blanket" if the floc concentration is sufficiently high. The height at which this blanket will remain suspended in the unit will depend on the velocity of flow through the blanket without damaging the structural integrity of the blanket. The fragility of the floc is such that it is not possible in normal operation (without some destruction of the floc that forms the blanket and a resultant decrease in efficiency of operation) to increase the upflow velocity to the point where the blanket would be lifted to the surface.

Eqs. 9 and 10 are valid only if the units are properly chosen. The total weight of the settling solids, expressed in Eq. 9, has two variable factors, A and V . The settling area is usually taken as the surface area of the settling section of the solids contact unit. This is correct only if the solids are distributed uniformly over this area which will be a factor of the inlet design. The settling velocity of the floc is the subsidence rate of the sludge blanket. This velocity is affected by the floc structure, which in turn is affected by the temperature of water, the concentration of solids to be removed, the concentration of chemicals added, the mineral content of the water, and the pH of the water. As a result of the nature of the floc, plain settling curves will not give the true settling velocity for Eq. 9.

Transformation of Eqs. 9 and 10 into actual designs involves many practical problems. As stated by Mr. Kalinske, the type of solids contact unit shown in Fig. 5 is dependent on the flow through the unit for maintaining the solids level. The unit shown in Fig. 6 overcomes this by recirculating the slurry. This type of unit affords the additional advantage of introducing the solids into the settling unit with a definite downward velocity and thus utilizing the density currents to aid in settling. However, the unit shown in Fig. 6 has the disadvantage of not distributing the solids uniformly over the entire settling area. This disadvantage is minimized by increasing the depth at which the solids enter the settling section. Thus, at best, the solids contact units will be (like most units in water and sewage disposal systems) a compromise.

The author has attacked rectangular sedimentation basins as being of poor design. Mr. Kalinske suggests increasing the efficiency by increasing the depth of the sedimentation basin. This is contrary to the ideas of Mr. Camp, which have been incorporated into the water treatment plant at Cambridge, Mass. At Cambridge a conventional sedimentation basin was used to increase the sedimentation efficiency by adding two trays at $\frac{1}{3}$ and $\frac{2}{3}$ of the original depth. Thus the sedimentation efficiency was increased by reducing the depth rather than by increasing it.

On the basis of empirical design (and most water and sewage clarification units are so designed) there is a definite need for further research on the mechanisms involved. Sound theoretical concepts combined with practical design and operation will eventually yield the most efficient solids contact units as well as other types of solids separation units.

A. A. KALINSKE,¹⁰ M. ASCE.—The writer has attempted to indicate some

¹⁰ Director of Research and Development, Infico Inc., Tucson, Ariz.

of the basic physical problems entering into the design of solids-contact basins. Messrs. Anderson and McKinney both state that the general relationships presented by the writer require much data regarding the settling characteristics of suspensions before an engineer can use such relationships in his design work. These data were not presented because they are nonexistent. Such data are dependent (as Mr. McKinney mentions) on so many other factors that mere tabulation for one set of conditions would serve no useful purpose. It was shown that such data should be obtained directly by relatively simple laboratory tests for the specific type of flocculated suspension in the basin. Of course, if it is impractical to make such measurements directly, the designer must examine data obtained from similar types of suspensions.

Mr. McKinney criticizes the use of the theory of the settlement of suspensions of discrete particles to analyze the settlement of suspensions of flocculated particles. As both Messrs. Anderson and McKinney observe, the behavior of flocculated particles is complex; any attempt to formulate rational mathematical expressions for such behavior would unquestionably be useless for design purposes. When one encounters such complex phenomena, many simplifications must be made in any theoretical analysis. Such analyses are nonetheless useful as a starting point for systematizing empirical data.

It was carefully emphasized that accurate design information could be obtained best from laboratory settling tests of suspensions under conditions approximating those (for example) in an upflow treating unit. Hundreds of such subsidence tests have been performed on a wide variety of flocculated suspensions of different concentrations. Such data have been extremely useful in the design and the capacity rating of solids-contact units. Subsequent study of the operation of such units in the field has confirmed the basic soundness and practicability of using such data as design criteria.

Mr. McKinney states that "at low rates of flow, the quantity of floc formed is small," and adds that therefore the quantity of floc present in the suspension will be low. In a properly designed and operated solids-contact treating basin, the total quantity of floc present in the blanket or slurry pool should be practically independent of the flow rate. It is desirable (and possible) to keep the floc suspension at approximately a uniform concentration, irrespective of the

flow rate or the rate of floc production. This is accomplished by wasting solids from the suspension in proportion to their rate of production; in many instances, this is in proportion to the flow.

The operation of the solids-contact unit shown in Fig. 6 has been noted by Mr. McKinney, who comments on the fact that the downward-circulating slurry is aided in settling by a density-current effect. There is no settling of the solids in the circulating zone of this unit; this occurs only in the concentrators, which occupy a part of the clarification area. There is a density-current effect, however, which helps in keeping the moving slurry from intermixing with the lighter, clear liquid above. A solids-contact unit, so designed, does distribute the solids uniformly over the entire clarification area. The circulation of the slurry at a rate equal to several times the treating flow rate is effective in distributing the solids equally over the entire clarification area of the basin.

Mr. McKinney states that the writer "has attacked rectangular sedimentation basins as being of poor design." This is not completely accurate. The writer did indicate that rectangular basins without proper take-offs may require larger and deeper basins to compensate for poor outlet design. Regarding the use of "trays" in rectangular basins to increase their capacity, R. Eliassen, M. ASCE, in his discussion of Mr. Camp's paper, pointed out some of the practical operating and design problems in connection with such design. It should be realized that this paper was concerned only with the design of solids-contact basins, which by their nature operate differently from conventional, straight, flow-through basins.

the development of a new species. The first step in this direction is to determine the extent to which the various characters of the species are correlated. This can be done by means of a correlation coefficient, which measures the degree of association between two variables. The correlation coefficient ranges from -1 to +1, where -1 indicates a perfect negative correlation and +1 indicates a perfect positive correlation. The correlation coefficient can be calculated using the formula:

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

where x_i and y_i are the individual observations, and \bar{x} and \bar{y} are the mean values of the two variables. The correlation coefficient is a measure of the strength and direction of the linear relationship between the two variables. It is important to note that the correlation coefficient does not indicate causation, only correlation.

**DISCUSSION OF SNOW HYDROLOGY FOR
MULTIPLE-PURPOSE RESERVOIRS
PROCEEDINGS-SEPARATE 189**

RAY K. LINSLEY,¹⁶ A.M. ASCE.—The accumulation of snow in the moun-

¹⁶ Associate Prof. of Hydr. Eng., Stanford Univ., Stanford, Calif.

tains of the western United States is economically important. Without this snow the development of this area would have taken a far different and probably less favorable course. To the engineer, however, this snow constitutes a troublesome design factor. Many approximations and assumptions are necessary in any computation of the runoff rates to be expected from melting snow. Mr. Riesbol has performed a service in presenting a fine summary of snow-hydrology practices.

The hydrothermogram procedure outlined by the author (under the heading, "Application to Spillway Design: Inflow Design Flood Based on Seasonal Snow Melt") is an interesting commentary on the present (1953) status of snow hydrology. The method is an application of the "degree-day" approach in which temperature above some base value is taken as an index of the melting rate. It is similar in many respects to the procedure suggested by the writer¹⁶

¹⁶ "A Simple Procedure for the Day-to-Day Forecasting of Runoff from Snowmelt," by R. K. Linsley, *Transactions, Am. Geophysical Union*, Pt. 3, 1943, pp. 62-67.

and basically the same as that proposed by G. D. Clyde,¹⁷ M. ASCE.

¹⁷ "Snow-Melting Characteristics," by G. D. Clyde, *Bulletin No. 231, Utah Agri. Experiment Station*, Logan, Utah, 1931.

A great deal of fundamental research in the field of snow physics has been conducted. Many features of the subject are imperfectly understood but a skeleton theory exists which should permit one to go beyond the degree-day approach to include in the melting index such parameters as solar radiation, humidity, and wind.

Mr. Barnes did not use these parameters (see under the heading, "Application to Spillway Design: Inflow Design Flood Based on Seasonal Snow Melt") because the necessary data for their use on an engineering scale were not available to him. A region of heavy snow accumulation is a region of scanty meteorological and hydrologic data. The variability of meteorological and hydrologic elements in rugged terrain and the economic importance of the region suggest that a station density greater than the national average might be in order. Continued research will fill out the skeleton of knowledge regarding snow physics. With this knowledge the basic data required to utilize it should also be available.

The amount of snow melted from a given basin in any period of time depends on the area of the snow pack exposed to melting conditions. This area is bounded by the lower edge of the snow cover and by the upper limit of the melting temperatures. Neither of these boundaries is strictly an elevation contour. Meteorological data may suffice to define the level to which melting

temperatures extend, but the lower limit of the snow cover can be determined only by field location. Aerial photography offers one of the best means of obtaining these data, although terrestrial photography and reconnaissance mapping methods contribute to local determinations. It seems axiomatic that a rational computation of snow melt cannot be performed without some knowledge of the controlling area of the melting zone, just as the runoff from rainfall on a basin cannot be estimated without first knowing the drainage area.

It is necessary in some cases to vary both the base temperature for the hydrothermogram and the degree-day factor, or melting rate of runoff, as the season progresses. The variation in the base temperature results from the fact that the melting zone moves progressively higher above the temperature station as the melting proceeds. Similarly, the variation in the melting rate of runoff must result from the change in area of the melting zone. Other factors such as the seasonal variation in solar radiation are undoubtedly important, but there is clearly a need for data defining the critical melting zone.

In Fig. 1 is shown an analysis which has been applied to the computation of the snow-melt hydrograph of a small area. When this analysis is applied to the hydrographs of snow-melt runoff from large areas, it is often found that the recessions of the daily increments of snow melt intersect the falling limb of the total hydrograph. A more rational approach seems to visualize the total hydrograph as the sum of a hydrograph of direct runoff and a ground-water (or subsurface flow) hydrograph (Fig. 6(a)). This approach makes apparent the many problems which hamper the application of experience gained in the study of small basins to the engineering problems of large basins. It is quite probable that the development of techniques must wait for available data on large basins.

Under the heading, "Capacities of Multiple-Purpose Reservoirs: Runoff Forecasting Curves," Mr. Riesbol mentions only the use of snow surveys, whereas successful forecasts of seasonal runoff have been made using precipitation data recorded at regular stations of the Weather Bureau.¹⁸ Existing

¹⁸ "Recent Developments in Water Supply Forecasting from Precipitation," by M. A. Kohler and R. K. Linsley, *Transactions, Am. Geophysical Union*, 1949, pp. 427-436.

records of rainfall make possible forecasts of runoff for areas where snow surveys are not available. These records offer a better source of data for statistical analysis than do the snow-survey records. Since the data on precipitation are regularly available, the cost of the forecasts is less than the cost of forecasts made from snow surveys. It appears that the accuracy of the forecasts based on precipitation data is as good as those based on snow surveys, although the superiority of either method depends on the local area and the data available. Precipitation data for forecasts of this type certainly deserve consideration for use in snow hydrology.

R. W. GERDEL,¹⁹ M. ASCE.—An improved approach to the development

¹⁹ Chf., Field Operations and Analysis Branch, Snow, Ice and Permafrost Research Establishment, U. S. Army, Wilmette, Ill.

of operational procedures for multiple-purpose reservoirs in areas where precipitation in the form of snow is the dominant hydrologic element has been presented by Mr. Riesbol. The author has emphasized the fact that the hydrologic engineer is compelled to use empirical relationships to develop suitable design and operation criteria despite the considerable amount of research that has been conducted on snow-melt and runoff problems. The

gaps in the scientific literature concerning snow may be sufficiently large to justify the use of empirical relationships, but the use of hypotheses not compatible with experimental evidence or the known physical properties of snow is not justified.

The literature on snow hydrology, mechanics, and physics contains sufficient references to the liquid-water holding capacity of snow and to the metamorphic processes which occur in snow to refute any assumption that a snow pack must achieve a density of 40% before discharge of rain or snow-melt water will take place. Likewise, there is no basis for assuming that a snow pack will not continue to increase to a density of more than 40% without storage of liquid, or that it will not retain that increase in the form of the solid phase.

In 1948 the writer examined the existing pertinent literature in a search for information about the physical properties of a ripe, discharging snow pack, for a report to the International Commission on Snow and Glaciers.²⁰ It was found

²⁰ "Physical Changes in Snow-Cover Leading to Runoff, Especially to Floods," by R. W. Gerdel, Union géodésique et géophysique internationale Association d'Hydrologie Scientifique, Assemblée générale d'Oslo, Vol. II, Oslo, Norway, 1948, pp. 42-54.

that densities between 25% and 56.5% for a ripe, melting, actively discharging snow pack were reported by several investigators. A snow pack usually consists of a number of readily identifiable layers which are the products of separate storms. The variation in density between these layers may be large at the time of deposition and will be maintained even in the ripe pack. R. A. Work found that homogeneous density was never obtained in a ripe pack and that the density of different layers in a late-April, ripe snow pack varied between 45% and 58%.²¹ In the Central Sierra Snow Laboratory observers found variations

²¹ "Snow Layer Density Changes," by R. A. Work, *Transactions*, Am. Geophysical Union, Vol. 29, 1948, pp. 525-546.

of more than 10% between the densities of ripe, discharging, spring snow packs at different sample points in the 4-sq-mile basin.

The results of investigations of the discharge of water from a melting snow pack made by the Weather Bureau at the Soda Springs Cooperative Research Project, in California, showed that a pack of 52.8% density, rapidly melting under an air temperature of 65°F, had a total liquid and solid water-equivalent content of 14.8 in. at 3:30 p.m.²² This content consisted of 13.4 in. of water-

²² "Snow Studies at the Cooperative Snow Research Project, Soda Springs, California," *Annual Report*, U.S.W.B., 1943-1944, pp. 11-14.

equivalent snow crystals and 1.4 in. of liquid water. At 8:30 the following morning, when temperatures had been low enough during the night to prevent or retard melting, the pack density was still 53%, whereas the total water equivalent was only 13.8 in., consisting of 13.4 in. of water-equivalent snow crystals and 0.4 in. of liquid water. The studies showed that no liquid water was frozen in the snow during the night and that the 1-in. loss in water equivalent of the pack during the night could be accounted for by the drainage of the 1 in. of liquid water from the pack during the same period. The maintenance of almost constant density was due to shrinkage and reduction in voids.

The 1944-1945 report from this project presented the results of investigations of a heavy rain on new and old snow.²³ A new snow, five inches deep, and

²³ "Snow Studies at the Cooperative Snow Research Project, Soda Springs, California," *Annual Report*, U.S.W.B., 1944-1945, pp. 10-13.

of 10% density had been deposited on top of an old pack of 38.3% density. In places, the new snow was deposited on clean black top and concrete roadways.

During one day, 1.44 in. of rain fell upon the new snow. At the height of the storm the new snow, overlying the old snow, contained only 13% of liquid water. On the paved roadway, where water was impounded in depressions, the slush that developed had a free-water content of 30%. Under no circumstances could such a high free-water content be attributed to the water-holding capacity of the snow. It was definitely the result of snow floating on, or being submerged in, water impounded in a depression with an impervious bottom. It is of interest to note that when efforts were made to measure the density of the slush by the usual tube sampling and weighing procedure, the water drained out of the sample so rapidly that the maximum measured density was only 16.5%—much less than the 40% density assumed to be required before melt or rain water could be discharged from the pack.

During the period between 1943 and 1950, when the writer was actively engaged in snow-hydrology research in the western United States for the Weather Bureau and the Corps of Engineers, a number of experiments were performed to determine the effect of large amounts of water on the shrinkage and increase in density of monolithic cylindrical samples of snow. A cylindrical container equipped with a cutter was used to obtain snow core samples (8 in. in diameter and 24 in. long) from sensibly homogeneous horizons in the pack. A screen bottom was applied to the container, the sample was weighed, and was then saturated with water at 32°F. The core was returned to the pack to establish natural capillary tension through the screen bottom. At the end of approximately 4 hr, the core was removed from the container, carefully calipered and weighed, and the free-water content was measured by the calorimetric method. These studies showed that an 18%-density dry snow at a temperature of 32°F underwent a reduction in volume and thus increased its density to 26% with an unfrozen water content of 1.5%. During the shrinkage process more than 2 in. of the previously applied water drained out of the sample, but there was no loss of solids. A 24%-density snow was increased to a density of 31% by this process. Repeated addition of liquid water, equal to three times the water equivalent of the original solid snow crystals in the core sample, changed the density from 22% to 34% by reducing the volume while allowing more than 15 in. of liquid water to pass through the initial 24-in.-long core of snow. The water equivalent of the sample at the beginning and end of the experiment was 5.28 in. and 5.48 in., respectively. The positive increase of 0.2 in. in water equivalent probably represents the amount of liquid retained in the core.

The results of these studies have not been published heretofore because the investigations were limited to high mountain snow covers where steeply sloping land provided rapid drainage and because there was lacking the time and equipment to collect sufficient data with which to demonstrate the extent of application of the results. They are presented in this discussion in hope that the 40%-hypothesis may be subject to a revision based on experimental evidence rather than on empirical judgment.

In another paper²⁴ the writer has discussed some experiments performed on

²⁴ "The Transmission of Water Through Snow," by R. W. Gerdel, Minnesota International Hydraulics Convention, Univ. of Minnesota, Minneapolis, Minn., August 31-September 4, 1953.

the water transmission and retention capacity of snow as measured with an electronic capacitance meter developed for this type of investigation. It was

found that for snow of 35% to 46% density, water applied to the surface of the snow (in quantities of 0.5 in. to 2.0 in.) was transmitted through 0.9 in. to 24 in. of snow per min. This velocity appears to indicate a limited temporary detention capacity. The permanent retention capacity, when free drainage was available, was approximately 0.1 in. of water per ft of 50%-density snow. It was shown that the movement of water through the snow pack follows definite channels which develop perpendicularly and horizontally within the pack. These channels have been recognized by glaciologists who term them "firn pipes" or "glands." The channels develop into drains filled with coarse-grained crystals in the melting seasonal layer of snow and are similar to the rock-filled drains used in agricultural and other soil-engineering practices. When the ground is exposed as a melting snow pack recedes in the mountains, the flow channels which have developed at the contact between the snow pack and the ground are evident on some soils, as a dendritic pattern of erosion marks. When these channels develop over rock or less erodible soils, they appear as arched piping in the bottom snow layer.

There is little experimental evidence to indicate that a deep and dry snow cover will retain appreciable amounts of rainfall, as stated by the author. The presence of free water induces a rapid metamorphic process which reduces the surface area of the individual snow crystals to the minimum area compatible with the mass and stable form of the crystal. That is, the feather-like, or dendritic, snow flakes are converted to small crystals of hexagonal form which resemble a mass of table salt or rock salt. The surface detention capacity of the snow flake is rapidly reduced by the metamorphic process, and the water which may have been retained on the larger surface area is discharged. This metamorphic process is not uniform throughout the pack. The process appears to proceed more rapidly at one place than at another, a phenomenon which accounts for the development of the flow zones within the pack and at the ground surface.

The size of the metamorphosed grains in a spring snow pack is the same as the size of a mixture of fine and coarse sands, about 0.02 mm to 2.0 mm in diameter. A reference to the literature on soil moisture shows that a sand with such a size distribution and free drainage will hold from 1% to 3% of liquid water, the same quantity as has been measured and reported for a spring snow pack.

The temporary detention of liquid water in snow is a function of the time required for the development of internal flow channels; hydrologically, these channels are effective temporary supplemental tributaries to the normal drainage pattern of the basin. There is no evidence that snow must achieve a 40% density before this supplemental tributary system becomes operative nor that, having achieved this density, the supplemental tributary drainage can be considered as having a discharge capacity which is a function of the density in excess of 40%.

In presenting this discussion the writer does not wish to detract from the value of Mr. Riesbol's paper. This discussion is submitted so that hydrologists may give more recognition to the known physical and thermodynamic properties of snow and the associated hydraulic phenomena in developing snow-melt runoff formulas and design and operational criteria for multiple-purpose reservoirs.

As a source of information, attention is called to a bibliography of six thousand items on snow, ice, and frozen ground available in the Engineering Societies' Library, New York, N. Y.²⁵

²⁵ "Annotated Bibliography on Snow, Ice and Permafrost," *SIPRE Report 12*, Vol. III, Snow, Ice and Permafrost Research Establishment, Corps of Engrs., U. S. Army. Prepared by the Library of Congress, Washington, D. C., January, 1953.

HAROLD D. HAFTERSON,²⁶ A.M. ASCE.—One of the most important as-

²⁶ Chf., Hydrology Branch, Project Planning Div., Bureau of Reclamation, U. S. Dept. of the Interior, Region 1, Boise, Idaho.

pects of the multiple-purpose use of major reservoirs in the western and northern sections of the United States has been investigated by Mr. Riesbol. This subject was previously examined in a symposium—one of the papers of which was written by Albert L. Cochran,²⁷ M. ASCE.

²⁷ "Their Use for Flood Control," by Albert L. Cochran, from "Multiple-Purpose Reservoirs; A Symposium," *Transactions, ASCE*, Vol. 115, 1950, p. 818.

The ability to forecast the volume and shape of the flood-runoff hydrograph is a valuable asset in operating multiple-purpose reservoirs and in realizing the purposes which these reservoirs have been designed to meet. The author rightly states (under the heading, "Conclusions") that

"Great refinement in procedure, with corresponding economy in both design and operation, will result from * * * the extension of networks for the collection of data so that they include pertinent elements on a watershed basis."

Continual study of forecasting procedures and operating criteria—to utilize available information on snow depths, areal coverage, and melting processes—is being made (1953) by all reservoir-operating agencies in areas where snowmelt runoff provides a large part of the storable flow. Snow surveys are constantly being conducted to cover ever-widening areas. New methods of measuring depths and water content of the snow pack are being devised. Experiments are being continued to determine the relationship of snow melt to some of the more easily measured meteorological factors such as temperature, air movement, and relative humidity.

It is worthwhile to note that the development of forecasting procedures, and the application of resulting forecasts to the operation of multiple-purpose reservoirs, have provided methods whereby relatively full use of certain storage facilities can be made for functions which are ordinarily in opposition to each other. For example, in certain areas it is possible to use the full capacity of certain reservoirs both for flood control and irrigation, or for other conservation purposes, despite the fact that flood control requires reservation of reservoir space whereas irrigation demands the assured accumulation of storage for subsequent use.

Mr. Riesbol's use of the term "operation 'parameters'" might readily have been clarified by a definition of exactly what these parameters denote. The mathematical definition of a parameter is any constant or element whose values characterize one or more of the variables entering into a system of expressions or functions. Thus, the operating curves (which the author referred to under the heading, "Capacities of Multiple-Purpose Reservoirs") show the relationship between the storage space required to control the forecasted flood within specified limits of outflow and the volume of runoff which is anticipated between any given dates and the normal end of the flood season. The curves

shown in Fig. 2, empirically derived from floods of record, are curves that envelop the storage requirements for various volumes of total forecast runoff from any given date to July 31. An additional safety factor has been added to Fig. 2 to assure adequate reservation of space at the beginning of the flood period. This factor was added to account for the possible forecasting errors shown in the computations in Fig. 3.

To be more complete, Table 1 might have included a split period beginning on April 9 and continuing through April 16 so that the negative values could be shown to balance the positive values of required storage space. By means of this inclusion, the summations shown in Col. 5, Table 1, would be correctly referenced as to date.

The author's presentation of three methods for applying snow hydrology to spillway design includes the best of the available (1953) methods for deriving spillway-design floods of snow melt or combination rain and snow-melt types. It is gratifying to note that Mr. Riesbol suggests either the use of the unit-graph method or the augmentation of an actual snow-melt event to distribute the design snow-melt volume. Both procedures are practical and are superior to the widely used scheme of proportioning runoff directly to a design thermogram, ignoring the runoff characteristics of the basin.

It is felt that the detailed approach described in the second method (under the heading, "Application to Spillway Design: Flood Produced by Rain on Snow") is an admirable attempt to account for causative snow-melt factors other than air temperature. However, the number of assumptions necessary and the type of data required in the method prevent its practical use in most localities. Almost none of the numerous watersheds in the northwestern parts of the United States (for which spillway floods have been derived or will be derived by the USBR) has a temperature record within the watershed. The determination of values for K , U , and p in Eq. 3 and the determination of the required infiltration rates for such watersheds are highly speculative.

It is advantageous to use the hydrothermogram method for the derivation of spillway floods for large watersheds where seasonal snow-melt floods of several months' duration are prevalent. An examination of the method or methods for converting an actual and a design thermogram into corresponding runoff (through the hydrothermogram approach) would have been helpful to the potential user of this approach.

In determining the relationship between temperature and snow-melt runoff (Table 2), a correction should be made for the area contributing to snow melt during a given recorded event. In some cases, snow-course data can be used to correct continuously the contributing area during the given snow-melt flood. The author states (under the heading, "Application to Spillway Design: Flood from Snow Melt Alone")

"If the stream-gaging station is a considerable distance from the snow-melt area, it is necessary to determine time lag between snow-melting temperatures and runoff."

The location of the stream gage is only one of the factors which can cause a lag between temperature and runoff. The location of the temperature station with respect to the snow-melt area is another factor. A third factor is the time which is required for the heat in the air to convert the snow into water. The first two factors cause variance in the apparent time lag observed in

different snow-melt floods because of differences in the direction and speed of the respective causative warm-air mass. The third factor varies, depending on the depth and quality of the snow.

Should not the vertical axis of Fig. 4 be labeled, "Maximum fifteen-day melting rate of runoff, in inches per degree day" rather than "Maximum * * * of runoff, in inches?"

In the application of the detailed and somewhat theoretical method for deriving a design flood produced by rain on snow (Table 4), it appears that consideration should be given to the melting of the snow by the heat of the rain water. This amount of melting can be computed from

$$M = \frac{P(T_w - 32)}{144} \dots \dots \dots \quad (4)$$

in which M is the snow melt in inches, P denotes the precipitation in inches, and T_w is the wet-bulb temperature (assumed to be the temperature of the rain). Although generally regarded as insignificant, the heat in 7.2 in. of precipitation of the maximum 6-hr period at an assumed 50° F could cause 0.90 in. of additional snow melt in the example cited in Table 4—thus increasing the contribution to the mean-water excess of the basin from 4.70 in. to 5.60 in. for that single 6-hr period.

The relationship of snow melt to runoff has not been given as much consideration in the design and operation of multiple-purpose projects as its importance warrants. The qualitative use of available knowledge on this subject, applied through the judgment of the engineer, should be bolstered by continuous improvement of procedures to apply this increasing knowledge quantitatively.

DAVID M. ROCKWOOD,²⁸ A. M. ASCE.—The problems of snow hydrology

²⁸ Technical Director, Snow Investigations, Corps of Engrs., U. S. Dept. of the Army, North Pacific Div., Portland, Ore.

as related to the design and operation of water-control projects have been described by the author. Mr. Riesbol has also summarized methods for determining rates of snow melt, reservoir capacities, seasonal forecasts, design floods, and storage of liquid water in the snow pack.

The purpose of reservoir storage in the regulation of streams is twofold. This storage can be used for supplementing deficiencies in the natural flow for specified purposes, or it can be used for reducing overbank flows to specified channel capacities. In rivers where the occurrences of low and high flows invariably follow a seasonal pattern, the reservoir-storage space can be used to meet both purposes compatibly, provided proper operational procedures are established. In rivers where the stream flow is primarily the result of snow melt, and when there is normally a delay between the accumulation and melt periods, optimum use of storage space can be made because the volume of runoff during the melt period can be forecast two or three months in advance. The Columbia River in the Pacific Northwest is a notable example of a stream with these characteristics and is particularly susceptible to the development of multiple-purpose reservoir storage.

Efficiency of design and operation of multiple-purpose reservoir projects on snow-fed streams is dependent on the accuracy with which the stream flows can be forecast. In the matter of project design, the application of snow hydrology

involves determination of reservoir-storage requirements and the derivation of design floods to meet the criteria for spillways and appurtenances for the structure. For the operation of the project, knowledge of snow hydrology is a requirement for forecasting rates of flow for day-to-day operation, and also for volumetric-runoff forecasts in order to establish long-term operating criteria on a seasonal basis.

Engineers of the Corps of Engineers have long recognized the importance of snow hydrology for efficient design and operation of their projects, particularly those projects in the western United States. In 1945 the Corps of Engineers entered into a cooperative snow-investigation program with the Weather Bureau—with limited cooperation from other governmental agencies—for the purpose of determining criteria to be used in snow-hydrology problems. That program included (1) operation of three basin snow laboratories in the mountains of the western United States for periods of from 4 yr to 7 yr, (2) processing and publishing the hydrometeorological data from the laboratories, (3) maintaining an analytical unit for performing basic research in snow hydrology, and (4) disseminating information to participating offices. The cooperative phase of the program ended in 1952; however, the Snow Investigations Unit was continued by the Corps of Engineers with offices located (1953) in the North Pacific Division office. The activities of the unit were directed, almost entirely, toward specific analytical problems which were divided into two categories. The first was the analysis of physical processes affecting snow deposition and snow melt. The second was the practical application of procedures to basin snow-hydrology problems, which dealt with forecasting rates of stream flow and estimating seasonal volume of runoff.

The use of degree-day factors provides a quick method of approximating daily increments of snow melt from readily available meteorological data because air temperature is an index of heat transfer by several processes. The complexity and variability of these processes require that temperature-index methods be used with considerable judgment. This judgment can be obtained only by study and evaluation of the components of heat transfer for various meteorological and basin conditions. For example, on clear days the processes of heat transfer in a forest-covered area are quite different from those processes for open areas whereas the total daily energy exchange may be approximately equal. An air-temperature index in the forest area for this condition is much more reliable than for an open area because the air temperature is a less reliable index for solar radiation than for convective heat transfer and long-wave radiation from the forest canopy. Differences in meteorological factors, such as vertical and horizontal distribution of moisture and temperature of the air (together with the associated cloud layers, air drainage, and stability), are related to the general atmospheric circulation, which in turn produces changes in these properties with respect to time. All these properties affect the rate of heat exchange in a manner which cannot be defined by air temperature at only a few surface index stations. The problem of accuracy becomes a matter of economic necessity. The project design or operation must warrant the expenditure of time and effort to obtain the required basic data and to analyze the meteorological and basin effects for each day's melt. When the requirements are such that empirical day-degree relationships are adequate, application and limitations of the method should be understood.

The examination by Mr. Riesbol of spillway-design floods resulting from snow melt alone is based on an analysis of historical stream-flow and temperature records for a particular stream. Empirical melt rates were determined, and interpolation to design conditions was made on the basis of a maximum observed runoff and a maximum enveloping melt rate corresponding to that runoff. These values do not necessarily represent the maximum flood potential of the basin, when one considers such important factors as the maximum possible snow accumulation, the minimum basin losses, the most optimum flood-producing temperature sequence, the meteorological events leading to the greatest rate of heat transfer at the time of the flood peak, and the maximum snow cover that can reasonably be expected at the time of the peak. The simultaneous occurrence of these important flood-producing factors would probably lead to a much greater flood flow than that derived from the analysis of a relatively few years of stream-flow records—during which time it is quite unlikely that maximum flood-producing conditions have occurred.

Floods produced by rain on snow are more complex and less certain of analysis than those caused by snow melt alone. Eq. 3 is used to evaluate heat transfer by convection and condensation but does not account for heat transfer by solar or long-wave radiation. The application of Eq. 3 to basin analysis depends on an evaluation of the basin constant, K . In the example shown for the Sly Park Dam site, it was not explained how K was determined for this particular drainage basin. The use of Eq. 3 for this example is valid, inasmuch as short-wave radiation is negligible under these conditions, and net long-wave exchange is primarily a function of air temperature. However, it should be realized that, for the evaluation of total heat exchange, radiation ordinarily causes a large part of the total melt.

The retention of liquid water in the snow pack has been the subject of much speculation. Qualitatively, the pack has been considered a "sponge" capable of retaining large amounts of liquid water entering as a result of rainfall or surface melt, thereby reducing the rates of flow that would have resulted from the same excess water being deposited on saturated ground. Observations (1953) on the impervious snow lysimeter at the snow laboratory in the Central Sierras near Soda Springs indicate that the permanent retention of liquid water in a 100-in.-thick snow pack of 31% initial density during a 5-in. rainstorm was approximately 1 in., which included 0.3 in. as a result of the increase in temperature of the pack (isothermally) to 0°C. There was, in addition, transitory storage of liquid water in the pack, in the amount varying from 1 in. to 2 in., but this water drained out completely within 10 hr after cessation of rainfall and snow melt. In deriving the water-holding capacity of the snow, Mr. Riesbol assumed for Table 4 that the snow pack must reach a density of 40% before any water excess is available at the ground for contribution to the stream flow. From the observations at the Central Sierra Laboratory, runoff from the snow pack commenced after the pack density increased to 32.5%; the maximum density was 34%. In Table 4, it is not clear how the increments of snow melt shown in Col. 4 could total 11.0 in. when the initial water equivalent of the snow pack was 6.0 in. Also (considering the total storm period) there were 29.6 in. of rainfall and 6.0 in. of snow-water equivalent, of which 0.5 in. remained in storage in the pack at the end of the period and 1.5 in. were retained initially in the ground. Thus, a total of 33.6 in. was available at the ground surface during the storm period. The total water excess, which

is the sum of the increments listed in Col. 13, is 26.63 in. The loss then is 7.0 in. during the 54-hr period, which is equivalent to an average loss rate of 0.13 in. per hr. For design-flood conditions, the value appears to be rather high for this area. In general, the retention and detention of liquid water in the relatively shallow snow pack shown by this example appears to be too great.

HERBERT S. RIESBOL,²⁹ M. ASCE.—The writer is indebted to Messrs.

²⁹ Asst. Chf., Hydrology Branch, Project Planning Div., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

Gerdel, Hafterson, Linsley, and Rockwood for their constructive discussion and criticism of the paper. The purpose of the paper was to demonstrate the engineering application of scientific knowledge and data of snow hydrology to the planning, design, and operation of multiple-purpose reservoirs. Mr. Gerdel has urged that this application not include the use of hypotheses incompatible with " * * experimental evidence or the known physical properties of snow * * *." Mr. Hafterson emphasizes the fact that the application must be based on the judgment of the engineer, and that procedures must be continuously developed to bridge the gap between physical knowledge and engineering application. Each of the discussers had emphasized the need for new and improved methods of measurement and for more basic data. The writer agrees with these recommendations.

Mr. Gerdel has shown that, under a variety of conditions, the density of a ripe, melting, actively discharging snow pack can vary over a wide range. Undoubtedly, this is correct. However, the writer has made the conservative engineering assumption (under the heading, "Application to Spillway Design: Flood Produced by Rain on Snow") that " * * no release of water from the snow pack would occur until the snow attained a density of 40% * * *." This assumption was based on an examination of considerable data which has been corroborated by Walter T. Wilson, A.M. ASCE. Mr. Wilson has stated³⁰:

³⁰ "Storage and Movement of Liquid Water in a Snowpack," by Walter T. Wilson, U. S. Weather Bureau, Washington, D. C., October 12, 1953.

"The appended table gives average density as a function of number of days since establishment of the snow cover. The values given are based on a large mass of data including shallow and deep snow, short seasons and long seasons, high and low elevations and a wide range of other conditions. Of course, particular instances frequently vary considerably from the average values, but experience has shown the average values to be reliable * * *."

Mr. Wilson's table of the average-density trend contains the following information:

Time since beginning of snow cover, in days	Average density (percentage)
0.....	8
1.....	12
2.....	14
5.....	16
10.....	20
20.....	25
50.....	30
100.....	40
200.....	50

Most of the snow floods which are dealt with in the design and operation of multiple-purpose reservoirs occur within 100 days to 200 days after the beginning of the snow cover.

Mr. Gerdel has stated that the temporary detention of liquid water in snow is a function of the time required for the development of internal flow channels, and Mr. Hafterson states that the detention varies with the depth and quality of the snow. In this connection Mr. Wilson³⁰ presents the following information and practical formula for water movement through a deep snow pack:

"If a dense snowpack is deep enough to warrant routing, instead of merely lagging the rain or melt * * *, after its waterholding capacity has been satisfied the rates of rainfall or snowmelt may be expressed as a hydrograph of inflow to the pack at its upper surface. The outflow hydrograph at the snow-ground interface may be approximated by lagging the inflow and routing it by reservoir analogy. If the snow depth is d feet, the lag is $0.3d$, and the routing coefficient is $0.5d$, both in hours. Using the Muskingum notation, K in hours equals half the snow depth in feet, X is zero, and the hydrograph is translated a number of hours equal to three-tenths of the snow depth in feet."

Mr. Hafterson has shown that, in the derivation of a design flood produced by rain or snow, consideration should be given to the melting of snow by the heat of the rain water. As Mr. Hafterson states, this source of melting is usually regarded as being of minor significance; however, in the Sly Park study, it appears that the melt from this source might contribute considerably to the flood runoff. Usually, the rain forms at a higher elevation and lower temperature than is indicated by the surface dew point. Thus the rain would be subject to some evaporative cooling during its descent, and, as a result, when the rain reaches the surface, it might be at a temperature several degrees lower than the surface dew point. This lower temperature would reduce the melt potential.

Mr. Linsley notes the similarity between the hydrothermogram and his (1943) procedure and states that the method is basically the same as that offered by Mr. Clyde (1931). It is believed that any valid procedure for constructing the hydrograph of discharge from snow melt would necessarily resemble Mr. Linsley's procedure. However, the only similarity between the writer's procedure and Mr. Clyde's procedure for net yield is in the assumption of a constant volume of melt per degree day. Actually, a more general and more flexible procedure is now (1953) in use in which both the basic temperature and the discharge factor are considered to be variable functions of the accumulated percentage of the seasonal runoff volume. Methods have been developed by engineers of the USBR for constructing and adjusting the relationships among these functions, using only temperature and runoff data, so that a knowledge of the elevations, the area of the melting zone, and the lapse rate is not essential. Such data, if readily available, could perhaps be used to refine the procedure. Data concerning the wind velocity, humidity, and solar radiation could also be used to refine the procedure; however, these data are seldom available for design-flood studies.

The hydrothermogram procedure developed by Mr. Barnes differs from the methods cited by Mr. Linsley as follows:

1. Ground-water flow and direct runoff are computed independently and concurrently from separate hydrothermograms. Different lag times, depletion factors, base temperatures, and discharge factors are used for the two elements of flow.

2. The base temperature and discharge factor are considered to be variables and are determined from the accumulated percentage of the seasonal runoff.

3. The runoff volume generated each day is expressed directly as the maximum daily discharge in cubic feet per second.

4. The subsequent distribution of the runoff is accomplished by combining all the flows (of direct runoff or of ground water) that are in the channel and by applying the appropriate constant depletion factor to the total.

Mr. Rockwood is correct in stating that the methods presented by the writer for the derivation of the spillway design flood resulting from snow melt will not yield the maximum possible flood potential from that source. These methods are used as a rational procedure to obtain a maximum probable flood event which would have a magnitude somewhat less than the maximum possible event. This maximum probable flood is of sufficient magnitude to justify its selection for prudent design purposes. Snow floods developed by these procedures are considerably in excess of maximum experienced floods for the area of derivation.

Mr. Rockwood inquires how the increments of snow melt in Col. 4, Table 4, could total 11.0 in. when the initial water equivalent of the snow pack was 6.0 in. The values in Col. 4, Table 4, are melt potential computed from Eq. 3 and are used only as a method of distributing the depth of snow on the ground at the beginning and end of each 6-hr time increment (Col. 5 and Col. 6, Table 4) after accounting for the increase in density (Col. 7, Table 4) up to the 40% limit. Following the attainment of a pack density of 40%, the values shown in Col. 4, Table 4, reduce the waterholding capacity of the pack (Col. 8, Table 4) by equivalent amounts. Mr. Rockwood states that " * * * a total of 33.6 in. was available at the ground surface during the storm period." Actually, this amount should be 35.1 in. as the 1.5 in. shown at the head of Col. 12, Table 4, is the retention which was assumed to have occurred during a previous event. This assumption provides a reasonable starting point on the retention-time curve. The average loss rate over a period of 54 hr of approximately 0.16 in. per hr falls within the range of retention rates derived by the writer for the area from hydrograph analysis.

The cooperative snow-investigation program, referred to by Mr. Rockwood, has contributed greatly to the physical theory underlying snow hydrology and to the practical applications of that theory. The USBR has maintained a close contact with that program, including the contribution of approximately four man-years of technical assistance. As the results of the program are fully analyzed and presented, they will become even more valuable to all engineers engaged in the planning, design, and operation of multiple-purpose reservoirs that receive their inflow from snow-fed streams.

Corrections for Transactions.—Eq. 3 should be changed to read as follows:

$$D = 0.00184 K U [(t - 32) 10^{-0.0000156 Z} + 0.00578 (p - 6.11)]$$

PROCEEDINGS-SEPARATES

The technical papers published in the past twelve months are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 79 (1953)

JUNE: 194(CP) & (AT), 195(SM), 196(CP) & (AT), 197(HY), 198(ST), 199(EM), D-134(HY), D-141(HY).

JULY: ^a 200(SM)^b, 201(ST)^b, 202(EM)^b, 203(SM)^b, 204(AT)^b, 205(EM)^b, 206(ST)^b, 207(SA)^b, 208(SA)^b, 209(ST)^b, 210(SU)^b, 211(EM)^c, 212(SU)^b, 213(IR)^b, 214(HW)^b, 215(SM)^b, 216(ST)^b, 217(ST)^b, 218(ST)^b, 219(ST)^b, 220(SM)^b, 221(HW)^b, 222(SM)^b, 223(EM)^b, 224(EM)^b, 225(EM)^b, 226(CO)^b, 227(SM)^b, 228(SM)^b, 229(IR)^b.

AUGUST: 230(HY), 231(SA), 232(SA), 233(AT), 234(HW), 235(HW), 237(AT), 238(WW), 239(SA), 240(IR), 241(AT), 242(IR), 243(ST), 244(ST), 245(ST), 246(ST), 247(SA), 248(SA), 249(ST), 250(EM)^c, 251(ST), 252(SA), 253(AT), 254(HY), 255(AT), 256(ST), 257(SA), 258(EM), 259(WW).

SEPTEMBER: 260(AT), 261(EM), 262(SM), 263(ST), 264(WW), 265(ST), 266(ST), 267(SA), 268(CO), 269(CO), 270(CO), 271(SU), 272(SA), 273(PO), 274(HY), 275(WW), 276(HW), 277(SU), 278(SU), 279(SA), 280(IR), 281(EM), 282(SU), 283(SA), 284(SU), 285(CP), 286(EM), 287(EM), 288(SA), 289(CO).

OCTOBER: ^d 290(all Divs.), 291(ST)^c, 292(EM)^c, 293(ST)^c, 294(PO)^c, 295(HY)^c, 296(EM)^c, 297(HY)^c, 298(ST)^c, 299(EM)^c, 300(EM)^c, 301(SA)^c, 302(SA)^c, 303(SA)^c, 304(CO)^c, 305(SU)^c, 306(ST)^c, 307(SA)^c, 308(PO)^c, 309(SA)^c, 310(SA)^c, 311(SM)^c, 312(SA)^c, 313(ST)^c, 314(SA)^c, 315(SM)^c, 316(AT), 317(AT), 318(WW), 319(IR), 320(HW).

NOVEMBER: 321(ST), 322(ST), 323(SM), 324(SM), 325(SM), 326(SM), 327(SM), 328(SM), 329(HW), 330(EM)^c, 331(EM)^c, 332(EM)^c, 333(EM)^e, 334(EM), 335(SA), 336(SA), 337(SA), 338(SA), 339(SA), 340(SA), 341(SA), 342(CO), 343(ST), 344(ST), 345(ST), 346(IR), 347(IR), 348(CO), 349(SM), 350(HW), 351(HW), 352(SA), 353(SU), 354(HY), 355(PO), 356(CO), 357(HW), 358(HY).

DECEMBER: 369(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU)^e, 368(WW)^e, 369(IR), 370(AT)^e, 371(SM)^e, 372(CO)^e, 373(ST)^e, 374(EM)^e, 375(EM), 376(EM), 377(SA)^e, 378(PO)^e.

VOLUME 80 (1954)

JANUARY: 379(SM)^e, 380(HY), 381(HY), 382(HY), 383(HY), 384(HY)^e, 385(SM), 386(SM), 387(EM), 388(SA), 389(SU)^e, 390(HY), 391(IR)^e, 392(SA), 393(SU), 394(AT), 395(SA)^e, 396(EM)^e, 397(ST)^e.

FEBRUARY: 398(IR)^f, 399(SA)^f, 400(CO)^f, 401(SM)^e, 402(AT)^f, 403(AT)^f, 404(IR)^f, 405(PO)^f, 406(AT)^f, 407(SU)^f, 408(SU)^f, 409(WW)^f, 410(AT)^f, 411(SA)^f, 412(PO)^f, 413(HY)^f.

MARCH: 414(WW)^f, 415(SU)^f, 416(SM)^f, 417(SM)^f, 418(AT)^f, 419(SA)^f, 420(SA)^f, 421(AT)^f, 422(SA)^f, 423(CP)^f, 424(AT)^f, 425(SM)^f, 426(IR)^f, 427(WW)^f.

APRIL: 428(HY)^e, 429(EM)^e, 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)^e, 437(HY)^e, 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

a. Beginning with "Proceedings-Separate No. 200," published in July, 1953, the papers were printed by the photo-offset method.

b. Presented at the Miami Beach (Fla.) Convention of the Society in June, 1953.

c. Presented at the New York (N.Y.) Convention of the Society in October, 1953.

d. Beginning with "Proceedings-Separate No. 290," published in October, 1953, an automatic distribution of papers was inaugurated, as outlined in "Civil Engineering," June, 1953, page 66.

e. Discussion of several papers, grouped by divisions.

f. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1954

PRESIDENT

DANIEL VOIERS TERRELL

VICE-PRESIDENTS

Term expires October, 1954:
EDMUND FRIEDMAN
G. BROOKS EARNEST

Term expires October, 1955:
ENOCH R. NEEDLES
MASON G. LOCKWOOD

DIRECTORS

Term expires October, 1954:
WALTER D. BINGER
FRANK A. MARSTON
GEORGE W. McALPIN
JAMES A. HIGGS
I. C. STEELE
WARREN W. PARKS

Term expires October, 1955:
CHARLES B. MOLINEAUX
MERCEL J. SHELTON
A. A. K. BOOTH
CARL G. PAULSEN
LLOYD D. KNAPP
GLENN W. HOLCOMB
FRANCIS M. DAWSON

Term expires October, 1956:
WILLIAM S. LaLONDE, JR.
OLIVER W. HARTWELL
THOMAS C. SHEDD
SAMUEL B. MORRIS
ERNEST W. CARLTON
RAYMOND F. DAWSON

PAST-PRESIDENTS *Members of the Board*

CARLTON S. PROCTOR

WALTER L. HUBER

TREASURER
CHARLES E. TROUT

EXECUTIVE SECRETARY
WILLIAM N. CAREY

ASSISTANT TREASURER
GEORGE W. BURPEE

ASSISTANT SECRETARY
E. L. CHANDLER

PROCEEDINGS OF THE SOCIETY

HAROLD T. LARSEN
Manager of Technical Publications

DEFOREST A. MATTESON, JR.
Editor of Technical Publications

PAUL A. PARISI
Assoc. Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

FRANK A. MARSTON, *Chairman*

I. C. STEELE

GLENN W. HOLCOMB

ERNEST W. CARLTON

OLIVER W. HARTWELL

SAMUEL B. MORRIS